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Generalized Coherent States as Preferred States of Open Quantum Systems

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We investigate the connection between *quasi-classical (pointer) states* and *generalized coherent states* (GCSs) within an algebraic approach to Markovian quantum systems (including bosons, spins, and fermions). We establish conditions for the GCS set to become most *robust* by relating the rate of purity loss to an invariant measure of uncertainty derived from quantum Fisher information. We find that, for damped bosonic modes, the stability of canonical coherent states is confirmed in a variety of scenarios, while for systems described by (compact) Lie algebras stringent symmetry constraints must be obeyed for the GCS set to be preferred. The relationship between GCSs, minimum-uncertainty states, and decoherence-free subspaces is also elucidated.

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The quest for quantum states resembling classical behavior dates back to the early days of Quantum Mechanics, when Schrödinger identified the closest to classical states of a quantum harmonic oscillator [1]. Following the work by Glauber and Sudarshan [2, 3], canonical coherent states (CCSs) played a pervasive role across photon and atom optics ever since [4]. The distinctive properties enjoyed by CCSs are rooted in the algebraic structure of the harmonic oscillator Hamiltonian. For systems described by different algebras, CCSs are replaced by *generalized coherent states* (GCSs) [5]. Like CCSs, GCSs are *minimum uncertainty states* [6], admit a natural phase-space structure [5], and are temporally stable under Hamiltonian evolution [7]. GCSs are ubiquitous in Nature: Aside from optical and atomic physics, states of matter such as BCS superconductors or normal Fermi liquids, for instance, are typically described by GCSs [5]. Recently, GCSs have been characterized as un-entangled within the framework of generalized entanglement [8] – such a property being related to efficient simulatability of Lie-algebraic models of quantum computation [9].

While all of the above properties make GCSs excellent candidates for *quasi-classical* states, no real-world system is isolated from its environment, causing pure quantum states to rapidly deteriorate into mixtures. How can quantum properties survive in the macroworld? Acknowledging the role of the environment and characterizing classicality as an emergent property of *open* quantum systems lies at the heart of the decoherence program [10]. *Pointer states* (PSs), in particular, are distinguished by their ability to persist in spite of the environment, thus are the *preferred* states in which open systems are found in practice. PSs exhibit *minimum purity loss* [11]. Several authors confirmed that for harmonic systems CCSs emerge under fairly generic conditions [12], which naturally raises some fundamental questions: What is the relationship between minimum purity loss and minimum uncertainty? Do the GCS and PS sets coincide in general? To what extent can stability properties against decoherence explain the special status that GCSs have in Physics?

In this Letter, we address the above questions for Markovian quantum systems [13] described by a Lie algebra $\mathfrak{g}$. Using the concept of *quantum Fisher information* (QFI) from estimation theory, we derive an invariant measure of uncertainty, and establish under which conditions such a measure is proportional to the dynamical rate of purity loss – thereby obtaining a streamlined method to link PSs to GCSs of $\mathfrak{g}$. Besides extending earlier results on single- and multi-mode bosonic algebras, we devote special emphasis to (semisimple compact) Lie algebras describing quantum spins and fermion systems [5]. For irreducible representations (irreps), we find that more restrictive symmetry requirements than in the bosonic case must be obeyed for the full GCS set to be preferred. For reducible representations, we propose an explicit characterization of the generalized minimum-uncertainty states manifold, which allows us to compare with information preserving structures as introduced in quantum information science. While decoherence-free subspaces (DFSs) [14] are found to be natural multi-dimensional extensions of PSs and GCSs under appropriate conditions, genuine noiseless subsystems (NSs) [15] are not – further pointing to the distinction between preserved pure states and preserved information which is quintessential to the NS idea.

Dynamical-algebraic setting and purity loss.– Let S denote the open system of interest, defined on a (separable) Hilbert space $\mathcal{H}$. We assume that the state $\rho(t)$ of S evolves according to a quantum Markovian master equation, that is, in units $\hbar = 1$ [13],

$$\dot{\rho}(t) = \mathcal{L}[\rho(t)] = -i[H_S, \rho(t)] + \mathcal{D}[\rho(t)]\,,$$
$$\mathcal{D}[\rho] = \frac{1}{2}\sum_{\ell}\Big([L_\ell\rho, L_\ell^\dagger] + [L_\ell, \rho L_\ell^\dagger]\Big)\,, \qquad (1)$$

where $\mathcal{L}[\rho]$, $\mathcal{D}[\rho]$ denote the Lindblad generator and dissipator, respectively. Both the physical (renormalized) Hamiltonian H_S and the Lindblad operators L_ℓ are bounded and time-independent. In the so-called weak

coupling limit (WCL), the perturbation due to $\mathcal{D}[\rho]$ is assumed to be weak enough for the relaxation time τ_R to set the slowest characteristic time scale – both relative to the reservoir correlation time, $\tau_R \gg \tau_c$ (Born-Markov condition), and the system dynamical time scale, $\tau_R \gg \tau_S$ (enabling the rotating wave approximation to apply). The WCL effectively constrains the Lindblad operators to obey the condition

$$[H_S, L_\ell] = \lambda_\ell L_\ell \,, \;\; \forall \ell \,, \tag{2}$$

where $\lambda_\ell \in \mathbb{R}$ are related to the Bohr frequencies of H_S [13]. Because the WCL is the only rigorous constructive approach for arbitrary temperature, primary emphasis will be given here to evolutions obeying Eq. (2). In such a case, we define the *dynamical Lie algebra* $\mathfrak{g}$ *associated to* S as the minimal Lie algebra generated by the L_ℓ. In phenomenological applications (and also the high-temperature limit and/or non-stationary environments), (2) is often waived. When considering such cases, we explicitly include H_S among the generators of $\mathfrak{g}$.

PSs are quantum states suffering minimal decoherence over a range of dynamical time scales [11]. They may be found by minimizing the *average purity loss*,

$$\overline{\Pi}_{|\psi\rangle} \equiv \frac{\Delta\Pi(\tau)}{\tau} = \frac{1}{\tau}\int_0^\tau dt\, \dot{\Pi}_{|\psi\rangle}(t) \,, \tag{3}$$

with $\Pi_{|\psi\rangle}(t) = 1 - \mathrm{Tr}\{\rho^2_{|\psi\rangle}(t)\}$, $\rho_{|\psi\rangle}(t)$ satisfying Eq. (1), $\rho_{|\psi\rangle}(0) = |\psi\rangle\langle\psi|$, and $\tau > 0$ to be determined. By moving to the Heisenberg picture, the rate of purity loss becomes

$$\dot{\Pi}_{|\psi\rangle}(t) = 2\sum_\ell (\Delta L_\ell(t))^2 \,, \tag{4}$$

where, for a generic operator O, the *(quasi)variance* is $(\Delta O)^2_{|\psi\rangle} = ||(O - \langle O\rangle_{|\psi\rangle})|\psi\rangle||^2 \equiv (\Delta O)^2$.

In the WCL, it is legitimate to treat the effects of $\mathcal{D}$ to first order in time. By exploiting Eq. (2), this yields

$$L_\ell(t) = e^{t\mathcal{L}^\dagger}[L_\ell] \approx e^{itH_S} L_\ell e^{-itH_S} = e^{it\lambda_\ell} L_\ell \,, \tag{5}$$

and the (first order) rate of purity loss becomes *time-independent* and equal to the average purity loss:

$$\dot{\Pi}_{|\psi\rangle} = \overline{\Pi}_{|\psi\rangle} = 2\sum_\ell (\Delta L_\ell)^2 \,. \tag{6}$$

GCSs and invariant uncertainty.– CCSs correspond to the action of the so-called Heisenberg-Weyl group on the single-mode Fock vacuum: $|\eta\rangle = \mathtt{D}(\eta)|0\rangle$, where $\mathtt{D}(\eta) = \exp(\eta a^\dagger - \eta^* a)$ denotes the phase-space displacement operator constructed from the oscillator algebra $\mathfrak{h}_3 = \{\mathbb{1}, a, a^\dagger\}$. GCSs are a generalization of this construction specified by three inputs [5]: a dynamical Lie group $\mathcal{G}$, with associated Lie algebra $\mathfrak{g}$ [16]; a unitary irrep Γ of $\mathcal{G}$; and a normalized reference state $|\Lambda\rangle$. Following [5], we require $|\Lambda\rangle$ to have *maximum symmetry* [17]. The GCSs associated to $(\mathcal{G}, \Gamma, |\Lambda\rangle)$ are defined as "generalized displacements" of $|\Lambda\rangle$: $|\Lambda, \eta\rangle = \mathtt{D}(\{\eta\})|\Lambda\rangle$, $\mathtt{D}(\{\eta\}) \in \mathcal{G}$. Among GCSs of finite-dimensional semisimple Lie algebras, noteworthy examples include $\mathfrak{su}(2)$-spin GCSs (also known as atomic coherent states [18]) as well as N-fermion GCSs associated to $\mathfrak{so}(2N) = \mathrm{span}\{c_i^\dagger c_j, c_i^\dagger c_j^\dagger, c_i c_j \,|\, 1 \leq i, j \leq N\}$ (including, as mentioned, the ground state of BCS theory [5]).

It is well known that CCSs achieve the lower bound in the Heisenberg uncertainty, $\Delta x \Delta p \geq 1/2$ (in appropriate units). *Generalized uncertainties* set limits on the precision with which a given parameter may be estimated [19]. Such a precision is quantified using the QFI [20]. A parameter θ has generator K_θ if $\partial_\theta \rho_\theta = -i[K_\theta, \rho_\theta]$, ρ_θ being the quantum state. If ρ_θ is pure, the corresponding QFI is $\mathcal{I}_\theta = 4(\Delta K_\theta)^2$, and the Cramer-Rao inequality for unbiased estimation dictates that

$$\delta\theta^2 \cdot \mathcal{I}_\theta = \delta\theta^2 \cdot 4(\Delta K_\theta)^2 \geq 1 \,. \tag{7}$$

With $\theta = x$, $K_\theta = p$, the above standard Heisenberg uncertainty principle is recovered. With $\theta = t$, $K_\theta = H$, we obtain the time-energy uncertainty, whereas $\theta = \phi$, $K_\theta = n$ gives the phase-number uncertainty.

The extension of these ideas to multiparameter estimation becomes relevant for the purpose of connecting PSs to GCSs. Let $\vec{\theta} = (\theta_1, \ldots, \theta_n)$. For pure states, the QFI matrix $\mathcal{I}_{\vec{\theta}}$ is

$$[\mathcal{I}_{\vec{\theta}}]_{j,k} = 4\langle (K_{\vec{\theta}_j} - \langle K_{\vec{\theta}_j}\rangle)(K_{\vec{\theta}_k} - \langle K_{\vec{\theta}_k}\rangle)\rangle \,. \tag{8}$$

Then the quantity $\mathrm{tr}\, \mathcal{I}_{\vec{\theta}}/4 = (\Delta\mathcal{I})^2$ represents a *scalar* measure of uncertainty. The quantum Cramer-Rao inequality also holds in the multiparameter setting. For example, with $K_{\theta_1} = x$, $K_{\theta_2} = p$, $(\Delta\mathcal{I})^2 = (\Delta x)^2 + (\Delta p)^2$, which is minimized by CCSs (but not squeezed states). If the generators $K_{\vec{\theta}}$ form a normalized basis of $\mathfrak{g}$ (with respect to the Killing metric), we recover the *invariant uncertainty* identified by Delbourgo [6]. By a suitable change of basis, $(\Delta\mathcal{I})^2$ may always be expressed as a sum of variances of observables, $(\Delta\mathcal{I})^2 = \sum_j (\Delta X_j)^2$, with $X_j^\dagger = X_j$. Physically, $(\Delta\mathcal{I})^2$ quantifies the *global* amount of uncertainty of $|\psi\rangle$ in a way which is invariant under arbitrary transformations in $\mathcal{G}$. The following *state-independent* lower bound for $(\Delta\mathcal{I})^2_{|\psi\rangle}$ holds:

Theorem 1 (*Invariant uncertainty principle*): Let Γ be an irrep of a semisimple Lie algebra $\mathfrak{g}$ with highest-weight element $|\Lambda\rangle$ (a reference state for the GCSs of $\mathfrak{g}$). Then

$$(\Delta\mathcal{I})^2_{|\psi\rangle} \geq \sum_j k_j \langle \alpha_j, \alpha_j\rangle \,, \tag{9}$$

where $\Lambda = \sum_j k_j \alpha_j$ in terms of simple roots α_j. The lower bound is attained iff $|\psi\rangle$ is a GCS of $\mathfrak{g}$.

A complete proof of Theorem 1 is rather technical, and while a sketch is included in [21], we refer to [22] for full detail. Physically, notice that for a spin-J irrep of $\mathfrak{su}(2)$, Theorem 1 recovers the familiar uncertainty relation for

angular momentum GCSs, $(\Delta J)^2 = \sum_{a=x,y,z}(\Delta J_a)^2 \geq J$. Although $\mathfrak{h}_3$ is *not* semisimple, it is intriguing that invariant uncertainty relationships structurally similar to (9) emerge upon mapping $\mathfrak{su}(2)$ into $\mathfrak{h}_3$ via a standard Holstein-Primakoff transformation in the large-J limit, $(\Delta J)^2/J \mapsto (\Delta a)^2 + (\Delta a^\dagger)^2 = (\Delta x)^2 + (\Delta p)^2 \geq 1$.

GCSs vs PSs for irreducibly represented algebras.– We first derive the conditions for the emergence of multi-mode CCSs in damped bosonic systems [4], with $H_S = \sum_{i=1}^n \omega_i(a_i^\dagger a_i + 1/2)$, $\omega_i > 0$.

Theorem 2: CCSs coincide with PSs iff there is a Lindblad operator L_ℓ proportional to each a_i or $a_i^\dagger$, or to $\sum_i a_i$ or $\sum_i a_i^\dagger$ for degenerate modes.

Proof: If no frequency degeneracy occurs, let each L_ℓ be of the form $c_\ell^{(i)} a_i$, $d_\ell^{(i)} a_i^\dagger$ for $c_\ell^{(i)}, d_\ell^{(i)} \in \mathbb{C}$. From Eq. (6) and the fact that $(\Delta a_i^\dagger)^2 = (\Delta a_i)^2 + 1$, the rate of purity loss is $\dot{\Pi}_{|\psi\rangle} = \sum_{\ell,i}(|c_\ell^{(i)}|^2 + |d_\ell^{(i)}|^2)(\Delta a_i)^2$, up to irrelevant constant terms. CCSs are eigenvectors of a_i, and are therefore PSs. If degeneracies occur, a similar argument shows that the rate of purity loss contains both terms proportional to $(\Delta a_i)^2$, and cross terms $\langle a_i^\dagger a_j\rangle - \langle a_i^\dagger\rangle\langle a_j\rangle$. Both are minimized iff $|\psi\rangle$ is a CCS. ∎

Eq. (6) may be used to identify GCSs for bosonic Lie algebras other than $\mathfrak{h}_3$. Focus, without loss of generality, on a single mode. If $\{L_\ell\}$ includes *quadratic* bosonic operators, the corresponding GCSs are squeezed states [5], resulting from displacements $\mathtt{D}(\xi,\eta) = \exp(\xi^* a^2 - \xi a^{\dagger 2})\mathtt{D}(\eta)$, $\xi, \eta \in \mathbb{C}$. In general, only a *subset* of GCSs emerge as PSs in the WCL, those that minimize the corresponding scalar uncertainty $(\Delta\mathcal{I})^2$: e.g., CCSs are PSs if a^2 is among the Lindblad operators, whereas only $|0\rangle$ is stable if $a^{\dagger\, 2}$ and/or $a^\dagger a$ are also included.

Different scenarios may arise beyond the WCL. Provided that the effects of the dissipator $\mathcal{D}$ may still be treated to first-order ($\tau_R > \tau_S$), relaxing condition (2) allows the $L_\ell(t)$ in Eq. (5) to acquire a non-trivial time-dependence – causing $\dot{\Pi}_{|\psi\rangle} \neq \overline{\Pi}_{|\psi\rangle}$ in general. Let us consider the case where $L_\ell = c_\ell a + d_\ell a^\dagger$, which includes the quantum Brownian setting of [11]. As $L_\ell(t) \approx c_\ell e^{-i\omega t} a + d_\ell e^{i\omega t} a^\dagger$, Eq. (4) yields both time-independent contributions and terms oscillating with frequency $\pm 2\omega t$. The latter integrate out if robust PSs are sought by extremizing the average purity loss, Eq. (3) with $\tau \simeq 2\pi/\omega$, leading to $\overline{\Pi}_{|\psi\rangle} = \sum_\ell(|c_\ell|^2 + |d_\ell|^2)(\Delta a)^2$, up to constants. Thus, CCSs still emerge as PSs [23]. For intermediate times, the states minimizing $\dot{\Pi}_{|\psi\rangle}(t)$ may be shown to be squeezed states with *time-dependent* squeezing parameter, in agreement with earlier results [12].

Consider next Markovian evolutions characterized by an irreducible *semisimple* Lie algebra $\mathfrak{g}$. Paradigmatic examples are d-level systems obeying the quantum optical master equation [4]. The following result follows from a direct application of Eq. (6) and Theorem 1:

Theorem 3: GCSs of $\mathfrak{g}$ coincide with PSs *if* $\overline{\Pi}_{|\psi\rangle} = |\lambda|^2(\Delta\mathcal{I})^2$.

Theorem 3 applies, in particular, if the Lindblad operators constitute an orthonormal basis of $\mathfrak{g}$, up to a global constant $\lambda \in \mathbb{C}$. Furthermore, in this case the Lindblad generator is *ergodic* and *unital*, implying that all initial states thermalize [13], and that the purity decreases monotonically [24] – which puts the first-order approximation on a firmer ground. Since a semisimple algebra $\mathfrak{g}$ may be uniquely expressed as a direct sum of simple algebras, $\mathfrak{g} = \oplus_u \mathfrak{g}_u$, the requirement of strict proportionality between $\overline{\Pi}_{|\psi\rangle}$ and $(\Delta\mathcal{I})^2$ in Theorem 3 may be weakened by allowing different proportionality constants for each $\mathfrak{g}_u$. While mathematically this leads to a condition which is also *necessary* for all GCSs to be PSs, the basic physical requirement is unchanged: as minimum uncertainty of GCSs reflects their high degree of symmetry, enhanced stability against decoherence is only ensured provided that $\mathcal{D}$ shares, itself, this symmetry. An illustrative situation is a damped two-level atom with non-radiative dephasing [4]

$$\begin{aligned}\mathcal{D}[\rho] &= \gamma_1(\bar{n}+1)([\sigma_-\rho,\sigma_+] + \text{h.c.}) + \\ &+ \gamma_1\bar{n}([\sigma_+\rho,\sigma_-] + \text{h.c.})\Big] + \gamma_2\Big([\sigma_z\rho,\sigma_z] + \text{h.c.}\Big),\end{aligned} \quad (10)$$

where $\gamma_j > 0$, $\bar{n}$ is the thermal photon number, and $\sigma_{\pm,z}$ are Pauli operators. PSs are always GCSs in this case, however arbitrary GCSs are PSs only for high temperature ($\bar{n}+1 \approx \bar{n}$) and $\gamma_2 \approx \bar{n}\gamma_1$. For higher-dimensional generalizations of (10) PSs need not be $\mathfrak{su}(2)$-GCSs if the conditions of Theorem 3 are not fulfilled.

PSs for reducibly represented algebras.– States suffering no purity loss to *all* orders in time under Markovian dynamics have been identified within DFS theory [14]. On one hand, DFSs are perfect *pointer subspaces*. On the other hand, DFSs require a *reducible* action of $\mathfrak{g}$. Can we relate DFSs to uncertainty and GCSs?

Consider a reducible representation Γ of a compact Lie group $\mathcal{G}$, with $\Gamma = \oplus_\mu n_\mu \Gamma_\mu$, $\mathcal{H} = \oplus_\mu \mathcal{H}_\mu$ being the associated irrep and state space decompositions, respectively, and $n_\mu \geq 1$ counting the μ-th irrep multiplicity. Theorem 1 allows an explicit characterization of *minimum invariant uncertainty states* to be given starting from the individual irreps: If Λ_μ is the highest weight of the μth irrep, the minimum of $(\Delta\mathcal{I})^2$ for Γ is achieved by the *minimum* highest weight. Let $|\Lambda\rangle$ be any normalized reference state in the $n_{\bar{\mu}}$-dimensional subspace generated by minimum highest weight vectors. Then the minimum-uncertainty manifold may still be formally obtained through a displacement of $|\Lambda\rangle$ – as such, minimum-uncertainty states retain *maximum symmetry*, although they are no longer the unique states with this property.

In the Markovian limit, a DFS subspace $\mathcal{H}_{\mathtt{DFS}}$ is defined by the property $\mathcal{D}(|\phi(t)\rangle\langle\phi(t)|) = 0$, for all $|\phi(t)\rangle \in \mathcal{H}_{\mathtt{DFS}}$ and all t. If the corresponding L_ℓ close a semisimple $\mathfrak{g}$ and the WCL is obeyed, $\mathcal{H}_{\mathtt{DFS}}$ consists of the set of states invariant under $\mathcal{G}$, that is, the *singlet* sector of $\mathfrak{g}$, $L_\ell|\phi\rangle = 0$ for all ℓ and $|\phi\rangle \in \mathcal{H}_{\mathtt{DFS}}$ [14]. Since the singlet corresponds to the irrep with the minimum highest weight, $(\Delta\mathcal{I})^2_{|\phi\rangle} = 0$ iff $|\phi\rangle \in \mathcal{H}_{\mathtt{DFS}}$, thus we have

Theorem 4: Let a DFS-supporting Markovian dy-

namics be described by a semisimple Lie algebra $\mathfrak{g}$. Then DFSs are minimum uncertainty pointer subspaces of $\mathfrak{g}$.

If GCSs are *defined* as minimum uncertainty states, the above Theorem immediately identifies DFSs with GCSs. However, GCS constructions for reducible representations do not retain all properties which GCSs enjoy for irreps (e.g., minimum-uncertainty GCSs need not be ground states of Hamiltonians in $\mathfrak{g}$). Regardless, neither the minimum-uncertainty or the GCS characterization extend to NSs, which provide the most general pathway to protecting quantum information [15, 25]. The key difference brought by the NS notion is the possibility that states in a *factor* $\mathcal{H}_{\mathtt{NS}}$ of a subspace of $\mathcal{H}$ are unaffected by $\mathcal{G}$ – while allowing *arbitrary evolution* in the full $\mathcal{H}$. Thus, NSs are not captured by the definition of PSs as pure preserved states of S adopted throughout. Since genuine NSs live in the multiplicity space of irreps with dimension higher than one, they necessarily carry *non-zero uncertainty.* We illustrate the DFS-NS comparison in a system of four spin-1/2s with $\mathfrak{g} = \mathfrak{su}(2)$ acting in $\mathcal{H} = (\mathbb{C}^2)^{\otimes 4}$ via the total spin representation (collective decoherence). The irrep decomposition reads $\Gamma_{1/2}^{\otimes 4} = \Gamma_2 \oplus 3\Gamma_1 \oplus 2\Gamma_0 \simeq \Gamma_2 \oplus (\mathbb{1}_3 \otimes \Gamma_1) \oplus (\mathbb{1}_2 \otimes \Gamma_0)$, $\mathbb{1}_n$ being the n-dimensional identity operator. The two-dimensional spin-0 sector $\mathcal{H}_0 \equiv \mathcal{H}_{\mathtt{DFS}}$. A three-dimensional NS lives in the spin-1 subspace, $\mathcal{H}_1 \simeq \mathcal{H}_{\mathtt{NS}} \otimes \mathcal{H}_{\mathtt{N}}$, where the noisy factor $\mathcal{H}_{\mathtt{N}}$ is also three-dimensional. For any $|\psi_1\rangle \in \mathcal{H}_1$, $(\Delta\mathcal{I})^2_{|\psi_1\rangle} \geq 1$, making the NS invisible to both the invariant uncertainty and the purity loss functionals.

Conclusions.– We have established Lie-algebraic conditions for the emergence of GCSs as PSs of Markovian quantum evolutions. Although our analysis rests on invoking purity as a measure of classicality, we expect that different criteria will agree as long as PSs are well defined [26]. Notably, spin GCSs have been recently identified as maximum *longevity* states against quantum reference frame degradation [27], suggesting the validity of similar conclusions beyond the Markovian regime. Yet, as NSs vividly exemplify, known measures such as purity loss seem too strong a criterion for sieving robust dynamical features in the presence of the environment. What weaker notion of a *pointer structure* can capture the robustness of observable properties of the system, for instance persistent *correlations* in otherwise non-robust states? From a condensed-matter standpoint, distinguishability of quantum states according to QFI-related indicators (including topological quantum numbers and Berry phases) is closely related to differential-geometric approaches to quantum phase transitions in matter [28]. How does this tie into GCSs and open-system theory? Ultimately, answering these questions will be relevant to synthesize and control novel, stable phases in interacting quantum systems.

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