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11–12 Gyr old white dwarfs 30 pc away

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ABSTRACT

We present a detailed model atmosphere analysis of two of the oldest stars known in the solar neighbourhood, the high proper motion white dwarfs SDSS J110217.48+411315.4 (hereafter J1102) and WD 0346+246 (hereafter WD0346). We present trigonometric parallax observations of J1102, which places it at a distance of only 33.7 ± 2.0 pc. Based on the state-of-the-art model atmospheres, optical, near-infrared, mid-infrared photometry and distances, we constrain the temperatures, atmospheric compositions, masses and ages for both stars. J1102 is an 11-Gyr-old (white dwarf plus main-sequence age), $0.62\,M_\odot$ white dwarf with a pure H atmosphere and $T_{\rm eff} = 3830$ K. WD0346 is an 11.5-Gyr-old, $0.77\,M_\odot$ white dwarf with a mixed H/He atmosphere and $T_{\rm eff} = 3650$ K. Both stars display halo kinematics and their ages agree remarkably well with the ages of the nearest globular clusters, M4 and NGC 6397. J1102 and WD0346 are the closest examples of the oldest halo stars that we know of.

Key words: stars: atmospheres – stars: distances – stars: individual: SDSS J110217.48+411315.4 – stars: individual: WD 0346+246 – stars: Population II – white dwarfs.

1 INTRODUCTION

The majority of the stars with an initial mass less than about $8\,M_\odot$ end up as white dwarfs (WDs). The short main-sequence lifetimes of intermediate-mass stars mean that they spend almost all of their lives as cooling WDs, radiating away their residual thermal energy slowly. To first order there is a simple relation between the age and luminosity of a WD (Mestel 1952). A typical $0.6\,M_\odot$ pure H atmosphere WD cools down to 3800 K in about 10 Gyr (Fontaine, Brassard & Bergeron 2001). Winget et al. (1987) and Liebert, Dahn & Monet (1988) were the first ones to use the oldest WDs in the solar neighbourhood to constrain the age of the Galactic disc. Further studies based on nearby high proper motion WDs demonstrate that the oldest disc WDs are about 8 ± 1.5 Gyr old (Leggett, Ruiz & Bergeron 1998; Harris et al. 2006; Kilic et al. 2010b, and references therein).

WD cosmochronology also constrains the ages of the oldest halo WDs in the nearest globular clusters or in the field. Hansen et al. (2004, 2007) use more than 100 orbit *Hubble Space Telescope* (*HST*) observations of M4 and NGC 6397 and derive ages of 12.1 (with a 95 per cent lower limit of 10.3 Gyr) and 11.5 Gyr, respectively. Another large *HST* programme on 47 Tucanae has recently been completed, but an age estimate from the WD cooling sequence is not yet available (Kalirai et al. 2012). The extension of

this method to halo WDs in the field is more problematic due to the unknown population membership of the high proper motion objects (see Oppenheimer et al. 2001b; Bergeron et al. 2005).

Fast moving halo WDs must exist in the solar neighbourhood, though in relatively small numbers compared to the larger population of younger disc WDs. For example, the Besançon Galaxy model predicts 127 disc and three halo WDs per square degree within 1 kpc for a Galactic latitude of $45°$ (Robin et al. 2003). However, the halo WDs are predicted to be fainter than $V = 22$ mag. Hence, deep, wide-field photometric and astrometric surveys are usually required to identify a significant population of field halo WDs. Such a survey based on the Sloan Digital Sky Survey (SDSS) + USNO-B (Munn et al. 2004) and the SDSS + Bok telescope proper motions identified several spectroscopically confirmed WD candidates (Kilic et al. 2010a,b). In addition, there are a dozen ultracool WDs that may be thick disc or halo WDs (e.g. Gates et al. 2004; Harris et al. 2008). However, trigonometric parallax measurements are required to confirm their halo membership. Large-scale surveys like the Panoramic Survey Telescope and Rapid Response System (Pan-STARRS), Palomar Transient Factory and the Large Synoptic Survey Telescope will be extremely useful for the identification of large samples of faint halo WDs (Tonry et al. 2012).

Even though the probability of finding a relatively nearby halo WD is small for any small patch of the sky, serendipitous discoveries of nearby halo WDs have happened. WD 0346+246 (hereafter WD0346) and J110217.48+411315.4 (hereafter J1102) are the best

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examples of such discoveries. Hambly, Smartt & Hodgkin (1997) identified WD0346 as a high proper motion WD (1.3 arcsec yr^{-1}) in the UK Schmidt Telescope photographic plates taken from 1987 to 1994. Similarly, Hall et al. (2008) identified J1102 as a high proper motion WD (1.75 arcsec yr^{-1}) while searching for high-redshift quasar candidates in the SDSS spectroscopy data (the SDSS spectrum of J1102 shows a broad emission feature at 5750–5900 Å, which is an artefact). Hall et al. (2008) explain the spectral energy distribution (SED) of J1102 with a 3830-K pure H atmosphere WD. However, without a parallax measurement, the mass, cooling age and the kinematic membership of this WD cannot be constrained accurately.

Here we present parallax and mid-infrared photometric observations of J1102, and a detailed model atmosphere analysis of both stars using parallax, optical and infrared photometry. Our observations are discussed in Section 2, whereas our model atmosphere analysis and the nature of these objects are discussed in Sections 3 and 4, respectively.

2 OBSERVATIONS

2.1 Parallax

We obtained trigonometric parallax observations of J1102 at the MDM 2.4-m Hiltner telescope. We used a total of 106 images taken on 13 observing runs between 2008 February and 2011 December. The instrumentation, observing protocols and reduction techniques used were very similar to those described in Thorstensen (2003), Thorstensen, Lépine & Shara (2008) and Kilic, Thorstensen & Koester (2008b). For nearly all observations we used a 2048^2 SITe CCD detector ('Echelle') at the f7.5 focus, but for the last two runs we used a 2048^2 STIS CCD ('Nellie'), due to problems with the Echelle CCD. Echelle has 24-μm pixels subtending 0.28 arcsec, and Nellie has 21-μm pixels subtending 0.24 arcsec. At each epoch we took many exposures in a 4-inch-wide Kron–Cousins I-band filter, as near to the meridian as we could to minimize differential colour refraction effects. The parallax reduction and analysis pipeline was unchanged from the previous work, and the change of detectors appeared to have no effect on the results.

For J1102, we measure a proper motion of $\mu = (-104.8 \pm 0.9, -1741.8 \pm 0.9)$ mas yr^{-1}, and a relative parallax and formal error of 28.8 ± 1.4 mas. Using the colours and magnitudes of the reference-frame stars, and increasing the uncertainty slightly to allow for unmodelled systematics, we estimate the absolute parallax to be 29.6 ± 1.7 mas. Our proper motion measurement is relative to the chosen reference stars. Corrections to absolute proper motions are generally of the order of 10 mas yr^{-1}. Our measurement is consistent with $\mu = (-106.9, -1750.0)$ mas yr^{-1} in the absolute reference frame from Munn et al. (2004).

Fig. 1 displays the trajectory of J1102 on the sky, and the same trajectory with proper motion component taken out. This figure shows that our observations cover a large range of parallax factor for J1102, and the parallax is well constrained. We use the full Bayesian formalism described in Thorstensen (2003) to estimate the distance, including the Lutz–Kelker correction and prior information from the proper motion and very liberal limits on the luminosity. J1102 is only 33.7 ± 2.0 pc away from us.

2.2 Mid-infrared photometry

We obtained *Spitzer* InfraRed Array Camera (IRAC; Fazio et al. 2004) 3.6-, 4.5-, 5.8- and 7.9-μm images of WD0346 and J1102
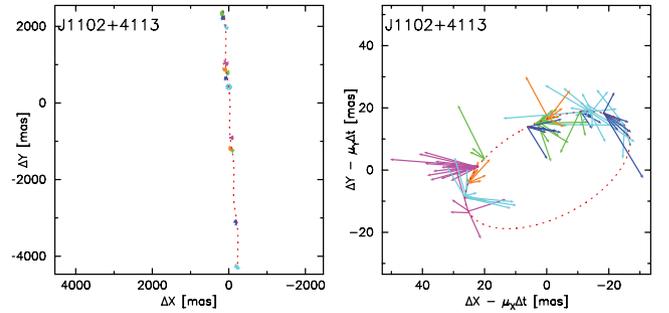
Figure 1. The trajectory of J1102 on the sky. Right-hand panel: the same trajectory with the proper motion taken out. The tip of each arrow is the position from a single image, and the tail is the computed location based on the fitted trajectory including zero-point, proper motion and parallax.

as part of the Cycle 3 programme 30208 and Cycle 5 programme 474. For each object, we obtained 100-s exposures for five dither positions in each filter. Our reduction procedures are similar to the procedures employed by Kilic et al. (2009b). Since our targets are relatively faint, we use the smallest aperture (2 pixels) for which there are published aperture corrections. For J1102 we measure 58.1 ± 2.3, 40.4 ± 3.0, 35.1 ± 9.8 and 23.3 ± 13.9 μJy in channels 1, 2, 3 and 4, respectively. Unfortunately, WD0346 is blended with a brighter source in the IRAC images, which prohibits us from performing aperture photometry on it. None of these objects is detected in the *WISE* survey (Wright et al. 2010).

3 MODEL ATMOSPHERE ANALYSIS

We use the state-of-the-art WD model atmospheres to fit the available photometry for our targets. The model atmospheres include the Lyα red wing opacity (Kowalski & Saumon 2006) as well as non-ideal physics of dense helium that includes refraction (Kowalski & Saumon 2004), ionization equilibrium (Kowalski et al. 2007) and the non-ideal dissociation equilibrium of H$_2$ (Kowalski 2006a). We convert the magnitudes into monochromatic fluxes using the zero-points derived from the Vega (STIS) spectrum integrated over the passband for each filter. We perform a two-dimensional least-squares fit in $T_{\rm eff}$ and the He/H ratio by minimizing the differences between the synthetic and the observed fluxes weighted by the observational errors. We use the parallax measurements to constrain the surface gravity.

3.1 J1102+4113

We use the optical and near-infrared photometry (Hall et al. 2008) plus our *Spitzer* photometry and distance measurement to model the J1102 SED. Fig. 2 presents our best-fitting model compared to the observations. J1102 is best explained by a $T_{\rm eff} = 3830$ K, $\log g = 8.08$ and pure hydrogen atmosphere WD with $M = 0.62$ M$_\odot$, which confirms the temperature assignment of Hall et al. (2008). The cooling age of the WD is $10.0^{+0.4}_{-1.1}$ Gyr. The error in the temperature estimate is about 200 K. The best-fitting model matches the infrared data fairly well; even though the H/He ratio is left as a free parameter in our model fits, the best-fitting model always has a pure hydrogen atmosphere composition for this star. The fit to the optical portion of the SED is reasonable, but not perfect, possibly due to the problems with the Lyα opacity calculations. The Lyα opacity calculations from Rohrmann, Althaus & Kepler (2011) do not result in any significant differences in our model calculations for this star. The optical portion of the SED favours a slightly cooler
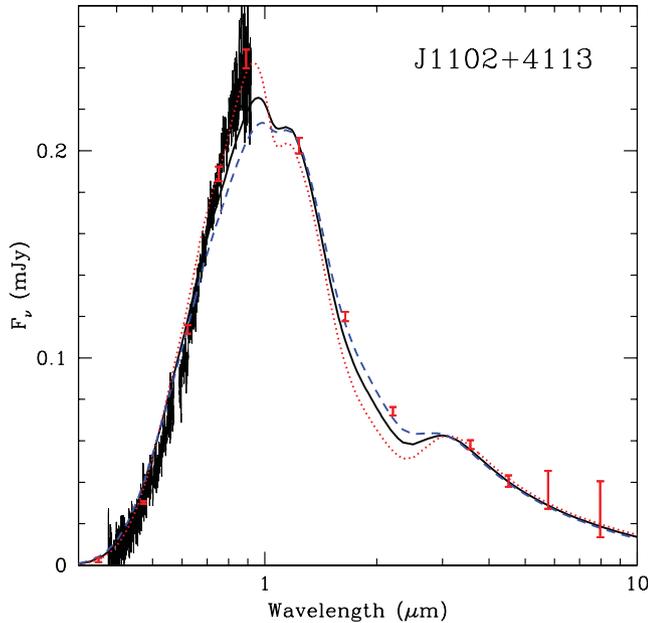
Figure 2. The J1102 SED compared to the best-fitting pure hydrogen atmosphere model spectrum ($T_{eff} = 3830$ K, solid line). Two other models with $T_{eff} = 3600$ K (dotted line) and 4000 K (dashed line) are shown for comparison. The optical spectrum is from the SDSS, with the artefact at 5750–5900 Å blocked out.

Figure 3. The WD0346 SED (Oppenheimer et al. 2001a) compared to the best-fitting model with $T_{eff} = 3650$ K and He/H = 0.43. Two other models with $T_{eff} = 3500$ K (pure H atmosphere, dotted line) and $T_{eff} = 3600$ K (He/H = 1, dashed line) are also shown.

solution, whereas the infrared SED favours a hotter solution. There are known problems with the H_2–H_2 and H_2–He collision-induced absorption (CIA; Frommhold et al. 2010) calculations in dense WD atmospheres. Even though the strong CIA feature around 2 μm is reproduced in our models, the predicted feature around 1 μm has never been observed in cool WDs. Regardless of these problems, our best-fitting model matches the overall SED fairly well. J1102 is clearly a very cool and old WD with a pure hydrogen atmosphere.

3.2 WD 0346+246

Hambly et al. (1999) measure a parallax of 36 ± 5 mas for WD0346, placing it at 28 ± 4 pc. The optical to near-infrared SED of WD0346 is kindly made available to us by B. Oppenheimer and is presented in Fig. 3. Using rather crude models for mixed H/He atmosphere WDs, Oppenheimer et al. (2001a) find a best-fitting solution of $T_{eff} = 3750$ K and a He-dominated atmosphere with trace amounts of hydrogen (log He/H = 9.1). Bergeron (2001) points out that an extremely helium rich composition is unlikely for such an old WD, because of accretion from the interstellar medium. In order to reproduce the short-wavelength end of the cool WD spectra, including WD0346, Bergeron (2001) adds a pseudo-continuum opacity (due to the bound-free opacity associated with the so-called dissolved atomic levels of the hydrogen atom). The spectrum of WD0346 is then fitted fairly well with a $T_{eff} = 3780$ K, log $g = 8.34$ and He/H = 1.3 model. More recently, Kowalski & Saumon (2006) and Kowalski (2006b) identified the missing opacity in the blue as the red wing of the Lyα absorption, and models including this opacity successfully reproduce the short-wavelength fluxes of many cool WDs (Kowalski & Saumon 2006; Kilic et al. 2008a; Kilic, Kowalski & von Hippel 2009a; Kilic et al. 2009b, 2010a; Durant et al. 2012; Giammichele, Bergeron & Dufour 2012).
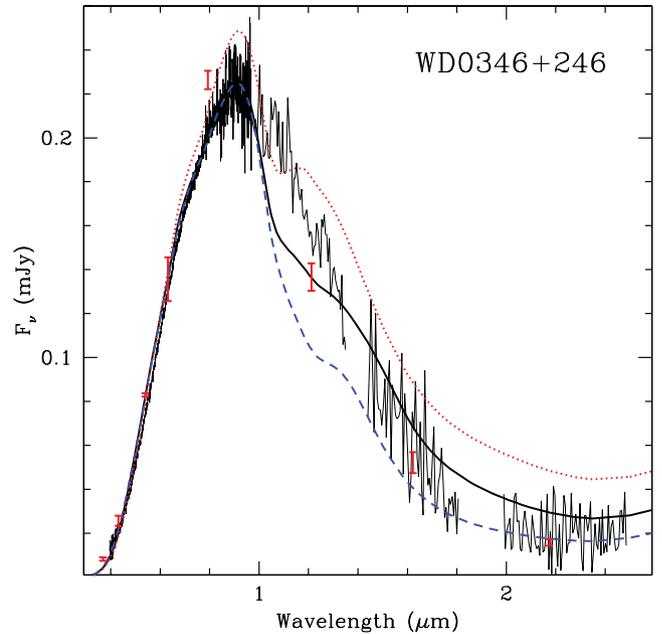
In this paper we use the models with the aforementioned Lyα opacity to derive the atmospheric parameters of WD0346. The observed SED of WD0346 is best explained by a mixed atmosphere model with $T_{eff} = 3650$ K, log $g = 8.3$ and He/H = 0.43, which is fairly consistent with the hydrogen-rich solution found by Bergeron (2001). Fig. 3 shows this model plus two other models with different H/He compositions. Unlike J1102, the pure hydrogen atmosphere model cannot reproduce the observed SED of WD0346 in the near-infrared. This is an indication that helium is present in the atmosphere and that through collisions with hydrogen species it enhances the CIA opacity. The best-fitting mixed H/He atmosphere model matches the optical portion of the SED extremely well; the temperature of the star is constrained accurately by the optical data. However, the fit to the near-infrared data is not perfect. As in J1102, the models predict a broad absorption feature around 1–1.2 μm that is not observed. Regardless of these issues, WD0346 is clearly a very cool and old WD. Our best-fitting model implies a mass of $0.77\,M_\odot$ and a cooling age of $11.2^{+0.3}_{-1.6}$ Gyr.

4 DISCUSSION

4.1 Total ages

Hall et al. (2008) classify J1102 as either a pure-hydrogen atmosphere WD at $T_{eff} = 3830$ K (the most probable solution) or a mixed H/He atmosphere WD with H/He $\approx 10^{-5}$ and $T_{eff} = 3500$ K. Thanks to our *Spitzer* photometry and parallax measurement, we are now able to constrain the atmospheric composition, mass and age of this WD. Our detailed model atmosphere analysis based on optical/infrared photometry and distance measurements for J1102 demonstrate that it is a 3830 K, $0.62\,M_\odot$ star with a WD cooling age of $10.0^{+0.4}_{-1.1}$ Gyr. The initial mass–final mass relation for WDs indicate that the progenitor of J1102 was a 1.8–$2.2\,M_\odot$ star (Catalán et al. 2008; Kalirai et al. 2009; Williams, Bolte & Koester 2009) with a main-sequence lifetime of 0.6–1.1 Gyr (Marigo et al.
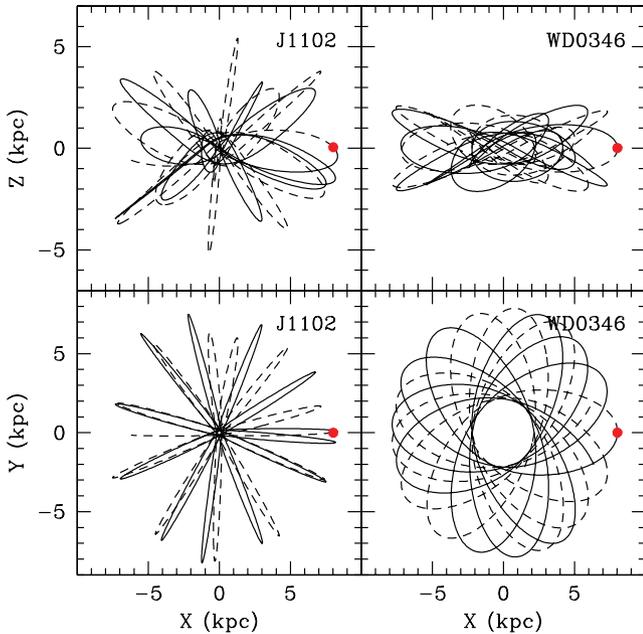
Figure 4. The Galactic orbits of J1102 and WD0346 for the past 1 Gyr (solid lines) and the next 1 Gyr (dashed lines). The current position of each WD is marked with a dot.

2008). Hence the total age of this object is 10.6–11.1 Gyr. Similarly, WD0346 is a 3650 K, 0.77 $M_\odot$ star with a WD cooling age of $11.2^{+0.3}_{-1.6}$ Gyr. The progenitor star was a 3.1–3.3 $M_\odot$ main-sequence star with a main-sequence lifetime of 240–270 Myr (Marigo et al. 2008). The uncertainty in main-sequence age is larger than the range given here. However, this uncertainty is insignificant compared to the total age of the WD, which is 11.5 Gyr for WD0346. Both WD0346 and J1102 are significantly older than the coolest disc WDs known (Leggett et al. 1998; Kilic et al. 2010b).

4.2 Halo membership

At a Galactic latitude of $+64°$, J1102 is 50 pc above the Galactic plane. Assuming zero radial velocity, it has $U = 68 \pm 5$, $V = -259 \pm 16$ and $W = 50 \pm 3$ km s^{-1} with respect to the local standard of rest (Schönrich, Binney & Dehnen 2010). The unknown radial velocity mostly affects the W velocity. For example, a change in the radial velocity from -50 to $+50$ km s^{-1} corresponds to a change in W from 5 to 95 km s^{-1} and V from -261 to -256 km s^{-1}. J1102 lags behind the Galactic disc. Fig. 4 plots the Galactic orbit of J1102 for the past and the next 1 Gyr, in a static disc–halo–bulge potential (Kenyon et al. 2008). J1102 goes above and below the plane by as much as 5 kpc, and its closest approach to the Galactic centre occurs at a distance of $R \approx 150$ pc. Even though J1102 is only 33.7 pc away from us right now, it will travel as far away as 16 kpc within the next 1 Gyr. J1102 is clearly a halo WD.

At a Galactic latitude of $-23°$, WD0346 is 9 pc above the plane. Assuming zero radial velocity, it has $U = -2 \pm 7$, $V = -148 \pm 22$ and $W = -55 \pm 9$ km s^{-1} with respect to the local standard of rest.[1] The unknown radial velocity mostly affects the U velocity. For example, a change in the radial velocity from -50 to $+50$ km s^{-1} corresponds to a change in U from $+43$ to -47 km s^{-1}. WD0346

also lags behind the Galactic disc. WD0346 goes above and below the plane by as much as 2.1 kpc and its closest approach to the Galactic centre occurs at a distance of $R \approx 2.2$ kpc. Even though WD0346 is only 28 pc away from us right now, it will also travel as far away as 16 kpc within the next 1 Gyr. WD0346 is most likely a halo WD.

4.3 The oldest stars in the solar neighbourhood

We now know of stars that are almost as old as the Universe (Hill et al. 2002). However, old stars with reliable age measurements are rare; radioactive elements like thorium and uranium are detected only in a few cases, and even then the age uncertainties are $\sim$2–3 Gyr (Frebel et al. 2007). Compared to main-sequence and giant stars, WDs provide an advantage in age measurements, thanks to the relatively short main-sequence lifetimes of typical 0.6 $M_\odot$ WDs and the well-understood physics of the cooling WDs. Sion et al. (2009) present a volume-limited sample of WDs within 20 pc of the Sun. Giammichele et al. (2012) show that the coolest WDs in that sample are about 4200 K; significantly hotter than the two WDs discussed in this paper.

With estimated ages between 11 and 12 Gyr, J1102 and WD0346 are currently the oldest stars known in the solar neighbourhood. The theoretical uncertainties due to the unknown core composition, helium layer mass, crystallization and phase separation are $\sim$1 Gyr for these ages (Montgomery et al. 1999). The age estimates for these two WDs are remarkably similar to the oldest WDs found in the nearest globular clusters M4 and NGC 6397. J1102 and WD0346 are about 100 times closer than these clusters. They provide an unprecedented opportunity to understand the model uncertainties in cool WD atmospheres and put the globular cluster ages on a more secure footing. Our analysis shows that the current WD atmosphere models including the Lyα opacity in the blue and the CIA in the infrared provide a reasonable match to the observations, though problems most likely associated with the CIA calculations remain. Infrared data are essential for constraining the atmospheric composition of cool WDs (see Figs 2 and 3). Hence, the temperature and age estimates based on two filter *HST* optical data are likely uncertain by a few hundred degrees and several hundred Myr, respectively.

5 CONCLUSIONS

We present trigonometric parallax and *Spitzer* mid-infrared photometric observations of the halo WD candidate J1102. We use these new data along with optical and near-infrared data on J1102 to constrain the temperature, mass and age of this WD. J1102 is an 11-Gyr-old WD 33.7 pc away from the Sun. We also revisit the model atmosphere analysis of another halo WD candidate, WD0346, using improved model atmosphere calculations. WD0346 is an 11.5-Gyr-old WD 28 pc away from the Sun. WD0346 and J1102 are currently the oldest stars known in the solar neighbourhood. Both stars display halo kinematics; they are just visiting our neighbourhood at the moment.

WD0346 and J1102 are the best examples of serendipitous discoveries of nearby halo WDs. Current and future deep, wide-field surveys ought to find many more old halo WDs. Such WDs remain to be discovered in Pan-STARRS, Palomar Transient Factory, the Large Synoptic Survey Telescope and *Gaia* data. These discoveries will provide independent age measurements and constrain the age and age range of the Galactic halo.

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[1] The *UVW* velocities for WD0346 were miscalculated by Kilic et al. (2010a) due to a sign error in its proper motion.

NOTE ADDED IN PROOF

After this paper was accepted, E. Wright pointed out that WISE J110217.41 + 411303.0 with W1 = 16.899 mag, or 54 ± 6 μJy is almost certainly J1102. This flux measurement is consistent with our IRAC channel 1 measurement within the errors. Hence it does not affect our conclusion.

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