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Abstract

The method of quantization of magnetic monopoles based on the order-disorder duality existing between the monopole operator and the lagrangian fields is applied to the description of the quantum magnetic monopoles of 't Hooft and Polyakov in the SO(3) Georgi-Glashow model. The commutator of the monopole operator with the magnetic charge is computed explicitly, indicating that indeed the quantum monopole carries $4\pi/g$ units of magnetic charge. An explicit expression for the asymptotic behavior of the monopole correlation function is derived. From this, the mass of the quantum monopole is obtained. The tree-level result for the quantum monopole mass is shown to satisfy the Bogomolnyi bound ($M_{\text{mon}} \geq 4\pi\frac{M}{g^2}$) and to be within the range of values found for the energy of the classical monopole solution.

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1) Introduction

A few years ago a general method of quantization of nonabelian magnetic monopoles was established [1] by exploiting the general fact that the operator which creates the topological excitations of a certain theory must also be dual to the basic lagrangian fields (in the sense of order-disorder duality) [2]. This method of quantization has been applied to a variety of systems containing topological excitations in two, three and four dimensional spacetime [3, 4, 5].

The nonabelian monopoles are topological excitations which occur when a nonabelian symmetry group (with compact covering) of a gauge theory is spontaneously broken down to a U(1) symmetry. The topological charge of the monopoles is the abelian magnetic charge corresponding to the unbroken U(1) [6]. As a consequence of the fact that monopoles are topological excitations appearing in a process of symmetry breakdown, it can be shown [1, 2] that for groups with a compact covering, the quantum creation operator of magnetic monopoles is the disorder variable for the phase transition in which the Higgs field develops a vacuum expectation value and thereby generates a mass to the gauge fields. In a Higgs phase, where $\langle\phi\rangle \neq 0$, we must have the vacuum expectation value of the monopole operator (μ operator) $\langle\mu\rangle = 0$. This automatically implies that μ creates states which are orthogonal to the vacuum, i.e., nontrivial states [2]. An explicit expression for the monopole operator in terms of the basic lagrangian fields of the theory is then constructed by imposing that it must satisfy an order-disorder algebra with these fields. Also a general expression for the correlation functions of these operators is obtained as an euclidean functional integral over the lagrangian fields by generalizing the methods first introduced by Kadanoff and Ceva for the description of correlation functions of disorder variables in the Ising model [8].

In the present work, we consider the magnetic monopoles of the SO(3) Georgi-Glashow model. We take the expression obtained in [1] for the quantum operator corresponding to the classical monopole solution and evaluate the long distance behavior of its two point correlation function by using the functional integral methods developed in [1]. We show that this correlation function decays exponentially and from its explicit expression the mass of the quantum monopole is obtained. The result generated in the lowest order in a loop expansion is found to be in agreement with the Bogomolnyi bound [9], $M_{\rm mon} \geq 4\pi \frac{M}{g^2}$ (M is the vector gauge field mass), for the value of the classical energy of the monopole solution.

In a recent publication [5] we performed the analogous computation in the case of vortices in the Abelian Higgs model in 2+1 D. There we also have found that at the tree level the quantum vortex mass coincides with the classical vortex solution energy, as obtained in [9], for example.

The quantum description of the excitations belonging to the topologically nontrivial sectors of the theory is of fundamental importance in order for a complete understanding of the system to be achieved. The obtainment of monopole correlation functions, is a basic step in the fullfilment of this goal. It would, on the other hand, be extremely interesting to investigate how much important are the quantum properties of monopoles in physical processes like the eventual monopole production in the early universe or in the catalysis of baryon decay [7]. We envisage these processes as interesting potential fields of application of the results of our work.

The organization of the paper is as follows. In Sec. 2 we give a brief description of the method of quantization of monopoles introduced in [1]. In Sec. 3 we implement the introduction of the external field in the functional integral describing the monopole correlation function, which is one of the key features of the method. In Sec. 4 the mass of the quantum monopole is obtained inlowest order in a loop expansion. The conclusions are presented in Sec. 5. Four Appendixes are included in order to demonstrate useful results.

2) The Quantization of Magnetic Monopoles

In this section, we are going to review the method of monopole quantization introduced in [1]. We also evaluate the commutator of the monopole operator with the magnetic charge and prove its nontriviality.

Let us consider the SO(3) Georgi-Glashow model, given by

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + \frac{1}{2}(D_\mu \phi)^a (D^\mu \phi)^a - \frac{1}{2} m^2 \phi^a \phi^a - \frac{\lambda}{4}(\phi^a \phi^a)^2 \, . \tag{2.1}$$

Throughout this work, we are going to use the adjoint representation where the generators of SO(3) are $(T^a)^{bc} = i\epsilon^{abc}$ $(a, b, c = 1, 2, 3)$, $[T^a, T^b] = i\epsilon^{abc}T^c$, $\mathrm{tr}T^aT^b = 2\delta^{ab}$. Then, the Higgs field belongs to an isospin triplet ϕ^a $(a = 1, 2, 3)$ and

$$G^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W^b_\mu W^c_\nu \, ,$$

$$(D_\mu\phi)^a = \partial\phi^a + g\epsilon^{abc}W^b_\mu\phi^c \, , \tag{2.2}$$

where g is the charge coupling constant. The model can exist in two phases according to whether $m^2 > 0$ or $m^2 < 0$. The first one is the symmetric phase and the second one is the "broken" phase where the Higgs field acquires a vacuum expectation value $|\langle\phi^a\rangle| = \varphi_0 = (|m^2|/\lambda)^{1/2}$. The theory possesses an identically conserved topological current which can be written as

$$J^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\partial_\nu\left[G^a_{\alpha\beta}\frac{\phi^a}{|\phi^a|}\right] \, . \tag{2.3}$$

By introducing the electromagnetic field

$$F^{\mu\nu} = G^a_{\mu\nu}\frac{\phi^a}{|\phi|} = \frac{1}{2}\mathrm{tr}G_{\mu\nu}\frac{\phi}{|\phi^a|} \, , \tag{2.4}$$

where $G_{\mu\nu} = G^a_{\mu\nu}T^a$, $W_\mu = W^a_\mu T^a$ and $\phi = \phi^a T^a$ we immediately realize that the topological charge density $J^0 = \frac{1}{2}\epsilon^{ijk}\partial_i F_{jk} = \vec{\nabla}\cdot\vec{B}$ is precisely the magnetic charge. Observe that the electromagnetic field $F_{\mu\nu}$ is gauge invariant: under $g(\alpha) = \exp[i\alpha^a T^a]$, we have $G_{\mu\nu} \to gG_{\mu\nu}g^{-1}$ and $\phi \to g\phi g^{-1}$.

It was shown by 't Hooft and Polyakov [6] that in the broken symmetry phase the theory admits classical finite energy solutions carrying a magnetic charge $4\pi/g$. The classical mass (energy) of the magnetic monopole solution was computed to be [6, 10, 11] $M^{\mathrm{cl}}_{\mathrm{mon}} = f(\frac{\lambda}{g^2})4\pi\frac{M}{g^2}$ where the pure number prefactor, $f(\frac{\lambda}{g^2})$, depends on the coupling constants ratio λ/g^2 ($f(0) = 1$ [10] and $f(\infty) = 1.787$ [11]). The monopole solution has the asymptotic behavior given by a nontrivial gauge transformation out of the vacuum $\phi^a_v = \varphi_0\delta^{a3}$, $W^a_{i,v} = 0$:

$$\phi^a(x) \overset{|\vec{x}|\to\infty}{\sim} \varphi_0\frac{x^a}{|\vec{x}|} = \varphi_0 g(\omega)^{ab}\delta^{b3} \, , \tag{2.5}$$

$$W^a_i(x) \overset{|\vec{x}|\to\infty}{\sim} -\frac{1}{g}\epsilon^{iab}\frac{x^b}{|\vec{x}|^2}$$

or

$$W_i = W_i^a T^a = -\frac{i}{g}\partial_i g(\omega) g^{-1}(\omega) \ , \tag{2.6}$$

where $g(\omega)$ is an SO(3) matrix with parameters given by [6]

$$\omega^a = \omega \quad \left(0, \frac{\tan\frac{\theta}{2}}{\sqrt{\sin^2\phi + \tan^2\frac{\theta}{2}}}, \frac{\sin\phi}{\sqrt{\sin^2\phi + \tan^2\frac{\theta}{2}}}\right) \ ,$$

$$\omega = 2\arccos\left(\cos\phi\cos\frac{\theta}{2}\right) \ . \tag{2.7}$$

The electromagnetic field associated with the asymptotic configuration, (2.5) and (2.6), is

$$F^{0i}_{\rm mon} = 0 \quad ; \quad F^{ij}_{\rm mon} = \frac{1}{g}\epsilon^{ijk}\frac{x^k}{|\vec{x}|^3} \ . \tag{2.8}$$

Since for the vacuum $F^{\mu\nu} = 0$, we see that under the transformation $g(\omega)$,

$$F^{\mu\nu} \stackrel{g(\omega)}{\longrightarrow} F^{\mu\nu} + F^{\mu\nu}_{\rm mon} \ . \tag{2.9}$$

The dual algebra which the magnetic monopole operator must satisfy is related to the above asymptotic solution [1]

$$\mu(x,S)\phi^a(y) = \begin{cases} g(\omega)^{ab}\phi^b(y)\mu(x,S) \ , & \vec{y} \in V_x(S) \\ \phi^a(y)\mu(x,S) \ , & \vec{y} \notin V_x(S) \end{cases} \tag{2.10}$$

$$\mu(x,S)W_i(y) = \begin{cases} \left[g(\omega)W_i(y)g^{-1}(\omega) - \frac{i}{g}\partial_i g(\omega)g^{-1}(\omega)\right]\mu(x,S) \ , & \vec{y} \in V_x(S) \\ W_i(y)\mu(x,S) \ , & \vec{y} \notin V_x(S) \ . \end{cases} \tag{2.11}$$

In the above expression S is a sphere of radius ρ, centered on $\vec{x}$ and $V_x(S)$ is the volume $\Re^3 - T(S)$ where $T(S)$ is the spherical volume bounded by S (Figure 1). Observe that as a consequence of the above algebra, μ is in principle a nonlocal operator depending on S. A local μ can be obtained in the limit when $\rho \to 0$.

An operator realization for μ can be obtained by using the external field [1]

$$\tilde{\mathcal{A}}_\mu(z;x) \equiv \tilde{\mathcal{A}}^a_\mu(z;x)T^a = -\frac{1}{g}\int_{V_x(S)} d^3\xi_\mu \omega^a(\xi - x)\delta^4(z-\xi)|_{\xi^0 = x^0} T^a \ . \tag{2.12}$$

In terms of this, we can write the monopole operator as [1]

$$\mu(x;S) = \exp\left\{\frac{-i}{2}\int d^4z \mathrm{tr}[D_\mu G^{\mu\nu}\tilde{\mathcal{A}}_\nu(z;x)]\right\} \ ,$$

$$\mu(x;S) = \exp\left\{-i\int d^4z[D_\mu G^{\mu\nu}]^a\tilde{\mathcal{A}}^a_\nu(z;x)\right\} . \tag{2.13}$$

Also, using the Yang-Mills equation

$$[D_\mu G^{\mu\nu}]^a = gj^{\nu a} \equiv g\epsilon^{abc}[D^\nu\phi]^b\phi^c , \tag{2.14}$$

we can write the μ operator as

$$\mu(x;S) = \exp\left\{-ig\int d^4z j^{\mu a}\tilde{\mathcal{A}}^a_\mu(z;x)\right\} . \tag{2.15}$$

As is shown in [1] these expressions for μ satisfy the dual algebra (2.10-2.11).

We can also compute the commutator of the monopole operator with the magnetic charge. Let us consider the local case in which the radius of the sphere S goes to zero. Using (2.10-2.11) or the more convenient form (for $\rho \to 0$)

$$\begin{aligned}
[\mu(x), W_i(y)] &= \left(\tilde{W}_i(y) - W_i(y)\right)\mu(x) ,\\
[\mu(x), \phi^a(y)] &= \left(\tilde{\phi}^a(y) - \phi(y)\right)\mu(x) ,
\end{aligned} \tag{2.16}$$

it is easy to show that

$$[\mu(x), G^a_{ij}(y)] = \left(\tilde{G}^a_{ij}(y) - G^a_{ij}(y)\right)\mu(x) . \tag{2.17}$$

In the above equations, $\tilde{\mathcal{O}}$ is the $g(\omega)$-transform of $\mathcal{O}$. Using (2.16-2.17) it is straightforward to show that

$$[\mu(x), F_{\mu\nu}(y)] = \left(\tilde{F}_{\mu\nu}(y) - F_{\mu\nu}(y)\right)\mu(x) , \tag{2.18}$$

or

$$[\mu(x), F_{\mu\nu}(y)] = F^{\rm mon}_{\mu\nu}\mu(x) , \tag{2.19}$$

where we used (2.9). The commutator of μ with the magnetic charge (topological charge) density is now immediately seen to be

$$\left[J^0(y), \mu(x)\right] = -\frac{1}{2}\epsilon^{ijk}\partial^{(y)}_i F^{\rm mon}_{jk}(y)\mu(x) = -\frac{1}{g}\nabla^2_{(y)}\left[\frac{1}{|\vec{x}-\vec{y}|}\right]\mu(x) = \frac{4\pi}{g}\delta^3(\vec{x}-\vec{y})\mu(x) . \tag{2.20}$$

This result shows explicitly that the operator μ does indeed carry $4\pi/g$ units of magnetic charge.

In ref. [1] a gauge equivalent form of the operator μ was used. Consider the set of parameters of an SO(3) transformation given by

$$\overline{\omega}^a = \theta\,(-\sin\phi, \cos\phi, 0)\,. \tag{2.21}$$

$g(\overline{\omega})$ shares with $g(\omega)$ the property (2.5), that is,

$$g(\overline{\omega})^{ab}\delta^{b3} = \frac{x^a}{|\vec{x}|}\,. \tag{2.22}$$

On the other hand, if we consider the configuration $\overline{W}_i = -\frac{i}{g}\partial_i g(\overline{\omega})g^{-1}(\overline{\omega})$, it follows that it must be gauge equivalent to W_i, Eq. (2.6), because both are gauge transforms of the vacuum ($W_{i,v} = 0$) [4]. This means that $\overline{W}_i = hW_ih^{-1} - \frac{i}{g}\partial_i hh^{-1}$, where $g(\overline{\omega}) = hg(\omega)$ and $h^{ab}\frac{x^b}{|\vec{x}|} = \frac{x^a}{|\vec{x}|}$. Another consequence is that the field intensity tensor configuration associated with $\overline{W}_i$ must be related to the one associated with W_i as $\overline{G}_{\mu\nu} = hG_{\mu\nu}h^{-1}$. It follows that the electromagnetic field obtained out of the vacuum ($F^{\mu\nu} = 0$) by $g(\overline{\omega})$ is

$$F_{\mu\nu} = 0 \xrightarrow{g(\overline{\omega})} \frac{1}{2}\mathrm{tr}hG_{\mu\nu}h^{-1}\hat{\phi}_r = \frac{1}{2}\mathrm{tr}G_{\mu\nu}\hat{\phi}_r = F_{\mu\nu}^{\mathrm{mon}}\,, \tag{2.23}$$

where $F^{\mu\nu}_{\mathrm{mon}}$ is given by (2.8), $\hat{\phi}^a_r \equiv x^a/|\vec{x}|$ and we have used the cyclic property of the trace as well as the fact that ϕ^a_r is invariant under h (or h^{-1}). As a consequence of (2.23) and (2.18-2.19) it follows that the operator μ constructed with $\overline{\omega}$ bears the same magnetic charge as the one constructed with ω. The operator $\mu(\overline{\omega})$ is gauge equivalent to $\mu(\omega)$. Troughout this work, we are going to use the operator μ expressed in terms of $\overline{\omega}$ because of its more convenient form.

The monopole operator μ is in principle nonlocal because it is defined on the volume (tridimensional hypersurface in four dimensional space) $V_x(S)$. A local operator, however, can be obtained by the introduction of an appropriate renormalization factor [1] and by taking the limit when ρ, the radius of S goes to zero. When computing correlation functions

[4] One of the authors (E.C.M.) is grateful to A. di Giacomo for calling his attention to the fact that the field configuration W_μ obtained by a gauge transformation of the vacuum with $g(\overline{\omega})$ is not identical to the one obtained with $g(\omega)$.

of μ, this is naturally done within the euclidean functional integral framework, by treating μ as a disorder variable and imposing hypersurface invariance on the expression for the correlation functions [1]. The hypersurface dependent renormalization counterterms appear then as self interactions of the external field $\tilde{\mathcal{A}}_\mu(z;x)$. Here we reproduce the final results for the hypersurface independent μ correlation functions and refer the reader to [1] for further details. The μ two point correlation function is given by (Euclidean space)

$$\begin{aligned}\langle\mu(x)\mu^\dagger(y)\rangle &= \lim_{\rho\to 0} Z^{-1}[0]\int DW_\mu D\phi D\eta D\overline{\eta}\exp\Big\{-\int d^4z\,\Big[-\frac{1}{8}\mathrm{tr}\,\{G_{\mu\nu}[W_\mu+\\ &+ \tilde{\mathcal{A}}_\mu(z;x,y)]\Big\}^2+\frac{1}{2}(D_\mu\phi)^a(D^\mu\phi)^a+V(\phi)+\mathcal{L}_{\rm GF}+\mathcal{L}_{\rm gh}\Big]\Big\}\;. \qquad (2.24)\end{aligned}$$

In this expression $\tilde{\mathcal{A}}_\mu(z;x,y)=\tilde{\mathcal{A}}_\mu(z;x)-\tilde{\mathcal{A}}_\mu(z;y)$, where $\tilde{\mathcal{A}}_\mu(z;x)$ is given by (2.12). $\mathcal{L}_{\rm GF}$ and $\mathcal{L}_{\rm gh}$ are the gauge fixing and ghost terms, respectively and η and $\overline{\eta}$ are the ghost fields. As is shown in [1], the above expression is hypersurface invariant and therefore local as a consequence of gauge invariance (or BRST invariance), because we can change the hypersurface $V_x(S)$ by means of a gauge transformation.

By making the change of variable $W_\mu\to W_\mu-\tilde{\mathcal{A}}_\mu(z;x,y)$ in the above functional integral we immediately obtain the equivalent form for the μ two point function:

$$\begin{aligned}\langle\mu(x)\mu^\dagger(y)\rangle &= \lim_{\rho\to 0} Z^{-1}[0]\int DW_\mu D\phi D\eta D\overline{\eta}\exp\Big\{-\int d^4z\,\Big[-\frac{1}{8}\mathrm{tr}\{G_{\mu\nu}\}^2+\\ &+ \frac{1}{2}(\tilde{\mathcal{D}}_\mu\phi)^a(\tilde{\mathcal{D}}^\mu\phi)^a+V(\phi)+\mathcal{L}_{GF}(W^a_\mu\to W^a_\mu-\tilde{\mathcal{A}}^a_\mu)+\\ &+ \mathcal{L}_{gh}(W^a_\mu\to W^a_\mu-\tilde{\mathcal{A}}^a_\mu)\Big]\Big\}\;, \qquad (2.25)\end{aligned}$$

where $\tilde{\mathcal{D}}_\mu=\mathbf{1}\partial_\mu-ig\left[W_\mu-\tilde{\mathcal{A}}_\mu(z;x,y)\right]$. By dropping the renormalization self interaction terms of the external field from (2.24) and (2.25) we can immediately recognize the expressions (2.13) and (2.15), respectively, for the monopole operator μ. These expressions are going to be our starting point for the obtainment of the long distance behavior of the monopole correlation function and mass. The great advantage of them is that their computation reduces to a standard computation in a field theory in the presence of the external field $\tilde{\mathcal{A}}_\mu$.

3) Introducing the External Field $\tilde{\mathcal{A}}_\mu^a(z)$ in the Broken and Symmetric Phases

In this section we study the introduction of the external field $\tilde{\mathcal{A}}_\mu^a(z)$, used in the description of the magnetic monopole correlation function, in both phases of the $SO(3)$ Georgi-Glashow model: (a) the symmetric phase, with the mass parameter in (2.1) $m^2 > 0$ (with $\langle\phi\rangle = 0$, for the vacuum expectation value of the Higgs field); and (b) the broken phase, with mass parameter $m^2 < 0$ ($\langle\phi\rangle \neq 0$).

3.1) Broken and symmetric phases

In the symmetric phase the Lagrangian density, $\mathcal{L}^S$, is just given by (2.1) and there is no need to make any shift in the Higgs fields around the vacuum. In the symmetric phase we have only to add to (2.1) a gauge-fixing term $\mathcal{L}_{GF}$ along with the corresponding ghost term $\mathcal{L}_{\rm gh}$. For the gauge-fixing term, we may choose a Lorentz-type gauge. Then, we add to (2.1), in the symmetric phase, the terms

$$\mathcal{L}_{GF}^S = -\frac{\xi}{2}\left(\partial_\mu W_a^\mu\right)^2 \ ,$$

$$\mathcal{L}_{\rm gh}^S = -\bar{\eta}^a\left(\delta^{ab}\Box - g\varepsilon^{abc}\partial^\mu W_\mu^c - g\varepsilon^{abc}W_\mu^c\partial^\mu\right)\eta^b \ , \qquad (3.1)$$

where η^a are ghost fields and ξ is the gauge parameter.

In the broken phase, $m^2 < 0$ in (2.1), the potential $V(\varphi^a\varphi^a) = \frac{m^2}{2}\varphi^a\varphi^a + \frac{\lambda}{4}(\varphi^a\varphi^a)^2$ has a minimum at $(\varphi^a\varphi^a) = \varphi_0^2$, with $\varphi_0^2 = \frac{|m^2|}{\lambda}$. Choosing the vacuum pointing along the third isospin axis, that is, $\varphi^a = \varphi_0\delta^{a\,3}$ and shifting the fields around this value, we see that the physical fields will be given by $(\phi_1, \phi_2, \phi_3) \to (\phi_1, \phi_2, \chi)$, with $\chi = \phi_3 - \varphi_0$. The Lagrangian density in the broken phase is then given, after shifting, by

$$\mathcal{L}^B = -\frac{1}{4}G^a_{\ \mu\nu}G^{a\,\mu\nu} + \frac{1}{2}\left[(\partial_\mu\phi_1)^2 + (\partial_\mu\phi_2)^2\right] + \frac{1}{2}\left[(\partial_\mu\chi)^2 - m_\chi^2\chi^2\right] +$$

$$
\begin{aligned}
&+ \quad \frac{M^2}{2}\left[(W_1^2)^2+(W_2^2)^2\right]+M\left[W_2^\mu(\partial_\mu\phi_1)-W_1^\mu(\partial_\mu\phi_2)\right]+g\left[W_1^\mu(\phi_2\partial_\mu\chi-\chi\partial_\mu\phi_2)+\right.\\
&+ \quad W_2^\mu(\chi\partial_\mu\phi_1-\phi_1\partial_\mu\chi)+W_3^\mu(\phi_1\partial_\mu\phi_2-\phi_2\partial_\mu\phi_1)]+\\
&+ \quad \frac{g^2}{2}\left[(W_1^\mu)^2(\phi^2+\chi^2+2\varphi_0\chi)+(W_2^\mu)^2(\phi_1^2+\chi^2+2\varphi_0\chi)+(W_3^\mu)^2(\phi_1^2+\phi_2^2)\right]+\\
&- \quad g^2\left[(W_1^\mu W_2^\mu)\phi_1\phi_2+(W_1^\mu W_3^\mu)(\phi_1\chi+\varphi_0\phi_1)+(W_2^\mu W_3^\mu)(\phi_2\chi+\varphi_0\phi_2)\right]+\\
&- \quad \frac{\lambda}{4}\left(\phi_1^2+\phi_2^2+(\varphi_0+\chi)^2\right)^2 , \qquad (3.2)
\end{aligned}
$$

where, in the expression above, $m_\chi^2 = 2\lambda\varphi_0^2$ and $M^2 = g^2\varphi_0^2$. The fields ϕ_1, ϕ_2 and χ have zero vacuum expectation value. In the broken phase we choose a R_ξ gauge (or 't Hooft gauge) [13], where the quadratic mixed terms involving $(W_\mu\phi)$ in (3.2) disappear. In the broken phase we have then for $\mathcal{L}_{GF}$ and $\mathcal{L}_{\rm gh}$ (adjoint representation) the expressions

$$
\mathcal{L}_{GF}^B = -\frac{\xi}{2}\left[\partial^\mu W_\mu^a + \frac{M}{\xi}\varepsilon^{ab3}\phi_b\right]^2 ,
$$

$$
(3.3)
$$

$$
\mathcal{L}_{\rm gh}^B = -\bar{\eta}^a\left[\delta^{ab}\Box - g\varepsilon^{abc}\partial^\mu W_\mu^c - g\varepsilon^{abc}W_\mu^c\partial^\mu + \left(\frac{M}{\xi}\phi_3+\frac{M^2}{\xi}\right)\left(\delta^{ab}-\delta^{a3}\delta^{b3}\right)\right]\eta^b .
$$

From Eqs. (2.1) and (3.1), the effective Lagrangian density in the symmetric phase is then given by $\mathcal{L}_{\rm eff}^S = \mathcal{L}^S + \mathcal{L}_{GF}^S + \mathcal{L}_{\rm gh}^S$, while in the broken phase, from Eqs. (3.2) and (3.3), the effective Lagrangian density is $\mathcal{L}_{\rm eff}^B = \mathcal{L}^B + \mathcal{L}_{GF}^B + \mathcal{L}_{\rm gh}^B$. From the quadratic terms in $\mathcal{L}_{\rm eff}^S$ and $\mathcal{L}_{\rm eff}^B$, we obtain the respective propagators for the fields. In Euclidean space these propagators are

$$
\begin{aligned}
\Delta_{(i)}(x) &= \int \frac{d^4k}{(2\pi)^4} e^{ik.x}\frac{1}{k^2+m_i^2} , \quad i=1,2,3 ,\\
D_{(1)}^{\mu\nu}(x) &= D_{(2)}^{\mu\nu}(x) = \int \frac{d^4k}{(2\pi)^4} e^{ik.x}\frac{1}{k^2+M^2}\left(\delta^{\mu\nu}-\frac{(\xi-1)k^\mu k^\nu}{\xi k^2+M^2}\right) ,\\
D_{(3)}^{\mu\nu}(x) &= \int \frac{d^4k}{(2\pi)^4} e^{ik.x}\frac{1}{k^2}\left(\delta^{\mu\nu}-\frac{(\xi-1)k^\mu k^\nu}{\xi k^2}\right) ,\\
\Delta_{\rm gh}^{(i)}(x) &= \int \frac{d^4k}{(2\pi)^4} e^{ik.x}\frac{1}{k^2+m_{{\rm gh}_{(i)}}^2} , \quad i=1,2,3 , \qquad (3.4)
\end{aligned}
$$

where $\Delta_{(i)}(x)$ are the propagators for the Higgs-field components, ϕ_1, ϕ_2 and ϕ_3 (χ in the broken phase), $D^{\mu\nu}_{(a)}(x)$ are the propagators for the gauge fields W^a_μ and $\Delta^{(i)}_{\rm gh}(x)$ are the propagators for ghost-field components. In the symmetric phase we have $M = 0$ and $m_i^2 = m^2$ and $m^2_{{\rm gh}_{(i)}} = 0$ ($i = 1, 2, 3$). In the broken phase we have $m_1^2 = m_2^2 = \frac{M^2}{\xi}$, $m_3^2 = m_\chi^2$ and $m^2_{{\rm gh}_{(1)}} = m^2_{{\rm gh}_{(2)}} = \frac{M^2}{\xi}$ and $m^2_{{\rm gh}_{(3)}} = 0$.

3.2) Introducing the external field in $\mathcal{L}_{\rm eff} = \mathcal{L} + \mathcal{L}_{GF} + \mathcal{L}_{\rm gh}$

As shown in Sec. 2, the two-point monopole correlation function $\langle \mu\mu^\dagger \rangle$ is given by Eq. (2.24), or, through a change of functional integration variable, $W_\mu \to W_\mu - \tilde{\mathcal{A}}_\mu$, by the equivalent form given by Eq. (2.25). We are going to use Eq. (2.25) for the evaluation of the two-point magnetic monopole correlation function. Let us write the exponent in (2.25) as $S_{\rm eff} = \int d^4z \left[\mathcal{L}^{\rm Eucl}_{\rm eff} + \tilde{\mathcal{L}}_{\rm eff}(\tilde{\mathcal{A}}_\mu)\right]$, where $\tilde{\mathcal{L}}_{\rm eff}(\tilde{\mathcal{A}}_\mu)$ contains all the dependence on the external field $\tilde{\mathcal{A}}_\mu(z; x, y)$.

In the symmetric phase, from Eqs. (2.1), (3.1) and (2.25), we obtain that (in Euclidean space)

$$
\begin{aligned}
\tilde{\mathcal{L}}^S_{\rm eff}(\tilde{\mathcal{A}}_\mu) \;=\;& -g\varepsilon^{abc}\tilde{\mathcal{A}}^b_\mu \phi^c \partial_\mu \phi^a + \frac{1}{2} g^2 \tilde{\mathcal{A}}^a_\mu \tilde{\mathcal{A}}^a_\mu \phi^b\phi^b - g^2 \tilde{\mathcal{A}}^a_\mu W^a_\mu \phi^b \phi^b + \\
& - \frac{1}{2} g^2 \tilde{\mathcal{A}}^a_\mu \tilde{\mathcal{A}}^b_\mu \phi^a\phi^b + g^2 \tilde{\mathcal{A}}^a_\mu W^b_\mu \phi^a\phi^b + \frac{\xi}{2}\left[\left(\partial_\mu \tilde{\mathcal{A}}^\mu_a\right)^2 - 2\partial_\mu \tilde{\mathcal{A}}^\mu_a \partial_\nu W^\nu_a\right] + \\
& + \bar{\eta}^a \left(g\varepsilon^{abc}\partial^\mu \tilde{\mathcal{A}}^c_\mu + g\varepsilon^{abc}\tilde{\mathcal{A}}_\mu \partial^\mu\right)\eta^b\,,
\end{aligned}
\tag{3.5}
$$

while in the broken phase, from Eqs. (3.2) and (3.3), we obtain (in Euclidean space)

$$
\begin{aligned}
\tilde{\mathcal{L}}^B_{\rm eff}(\tilde{\mathcal{A}}_\mu) =\;& \frac{M^2}{2}\left[\left(\tilde{\mathcal{A}}^\mu_1\right)^2 - 2\tilde{\mathcal{A}}^\mu_1 W^\mu_1 + \left(\tilde{\mathcal{A}}^\mu_2\right)^2 - 2\tilde{\mathcal{A}}^\mu_2 W^\mu_2\right] - g\left[\tilde{\mathcal{A}}^\mu_1\left(\phi_2\partial_\mu\chi - \chi\partial_\mu\phi_2\right) + \right.\\
& \left. + \tilde{\mathcal{A}}^\mu_2\left(\chi\partial_\mu\phi_1 - \phi_1\partial_\mu\chi\right) + \tilde{\mathcal{A}}^\mu_3\left(\phi_1\partial_\mu\phi_2 - \phi_2\partial_\mu\phi_1\right)\right] + \\
& + \frac{g^2}{2}\left[\left(\tilde{\mathcal{A}}^\mu_1\tilde{\mathcal{A}}^\mu_1 - 2\tilde{\mathcal{A}}^\mu_1 W^\mu_1\right)\left(\phi_2^2 + \chi^2 + 2\varphi_0\chi\right) + \right.\\
& + \left(\tilde{\mathcal{A}}^\mu_2\tilde{\mathcal{A}}^\mu_2 - 2\tilde{\mathcal{A}}^\mu_2 W^\mu_2\right)\left(\phi_1^2 + \chi^2 + 2\varphi_0\chi\right) + \left(\tilde{\mathcal{A}}^\mu_3\tilde{\mathcal{A}}^\mu_3 - 2\tilde{\mathcal{A}}^\mu_3 W^\mu_3\right)\left(\phi_1^2 + \phi_2^2\right) + \\
& - 2\left(\tilde{\mathcal{A}}^\mu_1\tilde{\mathcal{A}}^\mu_2 - \tilde{\mathcal{A}}^\mu_1 W^\mu_2 - \tilde{\mathcal{A}}^\mu_2 W^\mu_1\right)\phi_1\phi_2 - 2\left(\tilde{\mathcal{A}}^\mu_1\tilde{\mathcal{A}}^\mu_3 - \tilde{\mathcal{A}}^\mu_1 W^\mu_3 - \tilde{\mathcal{A}}^\mu_3 W^\mu_1\right)\left(\phi_1\chi + \varphi_0\phi_1\right) +
\end{aligned}
$$

$$
\begin{aligned}
&- 2\left(\tilde{\mathcal{A}}_2^\mu \tilde{\mathcal{A}}_3^\mu - \tilde{\mathcal{A}}_2^\mu W_3^\mu - \tilde{\mathcal{A}}_3^\mu W_2^\mu\right)(\phi_2\chi + \varphi_0\phi_2)\Big] + \frac{\xi}{2}\left[\left(\partial_\mu \tilde{\mathcal{A}}_a^\mu\right)^2 - 2\partial_\mu \tilde{\mathcal{A}}_a^\mu \partial_\nu W_a^\nu\right] + \\
&+ \bar{\eta}^a \left(g\varepsilon^{abc}\partial^\mu \tilde{\mathcal{A}}_\mu^c + g\varepsilon^{abc}\tilde{\mathcal{A}}_\mu^c \partial^\mu\right)\eta^b \,. \qquad (3.6)
\end{aligned}
$$

From Eq. (2.12) and the expression for the function $\varpi^a \equiv \varpi^a(\vec{z}-\vec{r})$, Eq. (2.21), since $\varpi^3(\vec{z}-\vec{r}) = 0$, we have $\tilde{\mathcal{A}}_3^\mu(z;x,y) = 0$ in Eqs. (3.5) and (3.6). From $\tilde{\mathcal{L}}_{\rm eff}^S(\tilde{\mathcal{A}}_\mu)$ and $\tilde{\mathcal{L}}_{\rm eff}^B(\tilde{\mathcal{A}}_\mu)$, we may extract the Feynman rules involving the external field. For example, at the tree-level order the relevant vertices are shown in Fig. 2.

4) The Monopole Correlation Function and Mass

4.1) The contribution at lowest order

Let us consider in this section the evaluation of the magnetic monopole two-point correlation function. Our starting point will be expression (2.25), as introduced in the last section. From (2.25), we can immediately see that

$$
\langle \mu(x)\mu^\dagger(y)\rangle = \exp\left\{\Lambda(x-y)\right\} \,, \qquad (4.1)
$$

where $\Lambda(x-y)$ is the sum of all Feynman graphs with the external field $\tilde{\mathcal{A}}_\mu^a(z;x,y)$ in the external legs.

In order to evaluate $\Lambda(x-y)$ we are going to use a loop expansion. In this work, we obtain the 0-loop result. In a forthcoming publication, we intend to consider the one-loop correction to this result [12].

We will be interested in the large distance behavior of (4.1), namely, when $|\vec{x}-\vec{y}| \to \infty$. As we show in Appendix A, only two legs graphs contribute to Λ in this limit. At 0-loop level, the two legs graphs containing the external field $\tilde{\mathcal{A}}_\mu^a$ are depicted in Fig. 3. Observe that the three last graphs of Fig. 3 only occur in the broken symmetry phase where the Higgs field possesses a nonzero vacuum expectation value φ_0 and the gauge field acquires a mass $M = g\varphi_0$ through the Higgs mechanism. In the symmetric phase, the contribution to $\Lambda(x-y)$ is given only by the first two graphs of Fig. 3. The sum of these graphs, in the symmetric phase, is easily seen to vanish by using the gauge field propagators given in Eq.

(3.4). This result immediately leads us to the conclusion that in the symmetric phase, where $\varphi_{_0} = 0$ and the additional graphs are absent, $\Lambda(x-y) \to 0$ at large distances, implying that

$$\langle \mu(x)\mu^\dagger(y)\rangle_S \stackrel{|\vec{x}-\vec{y}|\to\infty}{\longrightarrow} 1 \,. \tag{4.2}$$

This result implies that $\langle\mu\rangle \neq 0$ in the symmetric phase expressing the fact that the μ operator does not create states orthogonal to the vacuum not being, therefore, a truly monopole interpolating operator in this phase. This is an expected result in a phase where no classical monopole solution exists.

In the broken phase ($m^2 < 0$), from the graphs in Fig. 3 and the vertices in Fig. 2, we can write explicitly the asymptotic contribution to $\Lambda(x-y)$ as

$$\begin{aligned} \Lambda^{\rm asy}(x-y) &= \sum_{a=1}^{2} \int d^4z d^4z' \tilde{\mathcal{A}}^a_\mu(z;x,y) \left[-\frac{\xi}{2}\partial_\mu \partial'_\nu \delta^4(z-z') + \right. \\ &+ \frac{\xi^2}{2}\partial_\mu\partial_\alpha\partial'_\beta\partial'_\nu D^{\alpha\beta}_{(a)}(z-z') - \frac{M^2}{2}\delta^4(z-z') - \xi M^2 \partial_\mu\partial_\alpha D^{\alpha\nu}_{(a)}(z-z') + \\ &+ \left. \frac{M^4}{2} D^{\mu\nu}_{(a)}(z-z') \right] \tilde{\mathcal{A}}^a_\nu(z';x,y) \,, \end{aligned} \tag{4.3}$$

where $D^{\mu\nu}_{(a)}(z)$ is the Euclidean gauge propagator given in (3.4). Substituting $D^{\mu\nu}_{(a)}(z)$ in (4.3), we get the result

$$\Lambda^{\rm asy}(x-y) = -\frac{M^2}{2} \int d^4z d^4z' \tilde{\mathcal{A}}^a_\mu(z;x,y) \left[-\Box\delta^{\mu\nu} + \partial^\mu\partial^\nu \right] F(z-z') \tilde{\mathcal{A}}^a_\nu(z';x,y) \,, \tag{4.4}$$

where

$$F(z-z') = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik\cdot(z-z')}}{k^2+M^2} \,. \tag{4.5}$$

Observe that all the dependence on the gauge parameter ξ cancels and therefore (4.4) is completely gauge independent.

Let us observe now that in the adjoint representation, in which we are working, the internal indexes of ϖ^a (remember we are using the external field Eq. (2.12), with ϖ given by Eq. (2.21)) or $\tilde{\mathcal{A}}^a_\mu$ are of the same type as the spatial indexes. Since the spacetime index μ of $\tilde{\mathcal{A}}_\mu$ is always temporal because of our choice of the hypersurface $V_x(S)$ and since the indexes of $\overline{\varpi}^a$ are always spatial, we can write (4.4) as

$$\begin{aligned}\Lambda^{\rm asy}(x-y) &= -\frac{M^2}{2}\int d^4z d^4z' \tilde{\mathcal{A}}^\alpha_\mu(z;x,y)\left[-\Box(\delta^{\mu\nu}\delta^{\alpha\beta}-\delta^{\mu\beta}\delta^{\nu\alpha})+\delta^{\alpha\beta}\partial^\mu\partial^\nu-\right.\\ &- \left.\delta^{\alpha\nu}\partial^\mu\partial^\beta-\delta^{\mu\beta}\partial^\alpha\partial^\nu+\delta^{\mu\nu}\partial^\alpha\partial^\beta\right]F(z-z')\tilde{\mathcal{A}}^\beta_\nu(z';x,y)\,. \qquad (4.6)\end{aligned}$$

In order to write the last term between brackets in the above expression, we used the fact that $\vec{\nabla}\cdot\overline{\varpi}=0$. In Appendix B it is shown how to integrate by parts the derivatives ∂^α and ∂^β in order to make them to act on the $\overline{\varpi}$'s of the external field $\tilde{\mathcal{A}}_\mu$.

The expression between brackets in (4.6) can be written as

$$P^{\mu\nu\alpha\beta}=\epsilon^{\mu\alpha\rho\sigma}\partial_\sigma\epsilon^{\nu\beta\rho\lambda}\partial'_\lambda\,. \qquad (4.7)$$

Inserting this in (4.6), integrating by parts ∂_σ and ∂'_λ (see Appendix B) and eliminating the δ-functions appearing in the external fields, Eq. (2.12), we obtain

$$\begin{aligned}\Lambda^{\rm asy}(x-y) &= -\frac{M^2}{2g^2}\sum_{i,j=1}^{2}\lambda_i\lambda_j\int_{V_{x_i}}d^3\xi_\mu\epsilon^{\mu\alpha\rho\sigma}\partial^{(\xi)}_\sigma\overline{\varpi}^\alpha(\vec{\xi}-\vec{x}_i)\int_{V_{x_j}}d^3\eta_\nu\epsilon^{\nu\beta\gamma\lambda}\partial^{(\eta)}_\lambda\overline{\varpi}^\beta(\vec{\eta}-\vec{x}_j)\times\\ &\times\left[\delta^{\rho\gamma}\frac{1}{(2\pi)^2}(-\nabla^2_{(\xi)})\int_0^\infty dk\frac{e^{-\sqrt{k^2+M^2}|x^4-y^4|}}{\sqrt{k^2+M^2}}\frac{\sin k|\vec{\xi}-\vec{\eta}|}{k|\vec{\xi}-\vec{\eta}|}\right]\,, \qquad (4.8)\end{aligned}$$

where $x_1\equiv x$, $x_2\equiv y$, $\lambda_1\equiv +1$, $\lambda_2\equiv -1$. In the last expression, we also made the angular and k^4 integrations in $F(\xi-\eta)$, Eq. (4.5). Now, from our choice of the hypersurfaces V_{x_i}, the indexes μ and ν in (4.8) are temporal ($\mu=\nu=0$) and then we get

$$\begin{aligned}\Lambda^{\rm asy}(x-y) &= -\frac{M^2}{2g^2}\sum_{i,j=1}^{2}\lambda_i\lambda_j\int_{V_{x_i}}d^3\xi\,[\vec{\nabla}_{(\xi)}\times\overline{\varpi}]^l\int_{V_{x_j}}d^3\eta\,[\vec{\nabla}_{(\eta)}\times\overline{\varpi}]^m\times\\ &\times\left[\delta^{lm}\frac{1}{(2\pi)^2}(-\nabla^2_{(\xi)})\int_0^\infty dk\frac{e^{-\sqrt{k^2+M^2}|x^4-y^4|}}{\sqrt{k^2+M^2}}\frac{\sin k|\vec{\xi}-\vec{\eta}|}{k|\vec{\xi}-\vec{\eta}|}\right]\,. \qquad (4.9)\end{aligned}$$

Let us now make use of the following identity

$$\delta^{lm}\frac{\sin k|\vec{x}|}{|\vec{x}|} = \partial^l\partial^m\left(\frac{1-\cos k|\vec{x}|}{k}\right) + \frac{x^l x^m}{|\vec{x}|^2}\left(\frac{\sin k|\vec{x}|}{|\vec{x}|} - k\cos k|\vec{x}|\right) \ . \qquad (4.10)$$

Inserting this identity in the last term of (4.9), we get two terms: Λ_1 and Λ_2 :

$$\Lambda_1 = -\frac{M^2}{2g^2}\sum_{i,j=1}^{2}\lambda_i\lambda_j\int_{V_{x_i}} d^3\xi\,[\vec{\nabla}_{(\xi)}\times\overline{\omega}]^l\int_{V_{x_j}} d^3\eta\,[\vec{\nabla}_{(\eta)}\times\overline{\omega}]^m\ \times$$
$$\left[\frac{1}{(2\pi)^2}(-\nabla^2_{(\xi)})\int_0^\infty dk\frac{e^{-\sqrt{k^2+M^2}|x^4-y^4|}}{\sqrt{k^2+M^2}}\partial^l_{(\xi)}\partial^m_{(\eta)}\left(\frac{1-\cos k|\vec{\xi}-\vec{\eta}|}{k^2|\vec{\xi}-\vec{\eta}|}\right)|\vec{\xi}-\vec{\eta}|\right] \qquad (4.11)$$

and

$$\begin{aligned}\Lambda_2 \ &= \ -\frac{M^2}{2g^2}\sum_{i,j=1}^{2}\lambda_i\lambda_j\int_{V_{x_i}} d^3\xi\,[\vec{\nabla}_{(\xi)}\times\overline{\omega}]^l\int_{V_{x_j}} d^3\eta\,[\vec{\nabla}_{(\eta)}\times\overline{\omega}]^m\times\\ &\times\ \left[\frac{1}{(2\pi)^2}(-\nabla^2_{(\xi)})\int_0^\infty dk\frac{e^{-\sqrt{k^2+M^2}|x^4-y^4|}}{\sqrt{k^2+M^2}}\frac{(\vec{\xi}-\vec{\eta})^l(\vec{\xi}-\vec{\eta})^m}{k|\vec{\xi}-\vec{\eta}|^2}\left(\frac{\sin k|\vec{\xi}-\vec{\eta}|}{|\vec{\xi}-\vec{\eta}|}-\right.\right.\\ &-\ \left.\left. k\cos k|\vec{\xi}-\vec{\eta}|\right)\right] \ . \qquad (4.12)\end{aligned}$$

In the next two subsections, we are going to evaluate the contributions of each of them, respectively, to the long distance behavior of the magnetic monopole correlation function and mass.

4.2) The Λ_1 term

Let us consider here the contribution of (4.11) to (4.9). The derivatives $\partial^l_\xi\partial^m_\xi = -\partial^l_\xi\partial^m_\eta$ can be made total derivatives, because of they are contracted to rotationals. Then, using Gauss theorem we can write

$$\begin{aligned}\Lambda_1 \ &= \ \lim_{\rho\to\infty}\frac{M^2}{2g^2}\sum_{i\neq j;i,j=1}^{2}\lambda_i\lambda_j\oint_{S_i} d^2\xi_l\nabla^k_{(\xi)}[\vec{\nabla}_{(\xi)}\times\overline{\omega}(\xi-x)]^l\oint_{S_j} d^2\eta_m\nabla^k_{(\eta)}[\vec{\nabla}_{(\eta)}\times\overline{\omega}(\eta-y)]^m\times\\ &\times\ \frac{1}{(2\pi)^2}\int_0^\infty dk\frac{1}{\sqrt{k^2+M^2}}\left(\frac{1-\cos k|\vec{x}-\vec{y}|}{k^2|\vec{x}-\vec{y}|}\right)|\vec{x}-\vec{y}| \ . \qquad (4.13)\end{aligned}$$

In order to get (4.13), we integrated by parts the remaining derivatives in (4.9). We can show that the extra terms vanish in the limit $\rho \to 0$ (see Appendix D). We also considered $x^4 = y^4$ which correspond to taking the correlation function at equal real times when we make the analytic continuation back to Minkowski space. Observe that we already made $\vec{\xi} = \vec{x}$ and $\vec{\eta} = \vec{y}$ in the last part of the integrand. This is true in the local limit $\rho \to 0$. The additional terms in the Taylor expansion around $\vec{x}$ and $\vec{y}$ vanish in this limit. We have also dropped from (4.13) the unphysical self-interaction terms with $i = j$. These can be absorbed by a multiplicative renormalization of μ.

Remember (4.13) is actually valid only in the large distance limit ($|\vec{x} - \vec{y}| \to \infty$) since we are considering only the two-leg graphs contribution. Using the fact that

$$\lim_{|\vec{x}| \to \infty} \left[\frac{1 - \cos k|\vec{x}|}{k^2 |\vec{x}|} \right] = \pi \delta(k) \ , \tag{4.14}$$

we can easily perform the k-integral in (4.13). The surface integrals in (4.13) are evaluated in Appendix C. Combining the two results we get

$$\Lambda_1 = -\frac{\pi^5}{32} \frac{M}{g^2} |\vec{x} - \vec{y}| \ . \tag{4.15}$$

4.3) The Λ_2 term

Let us consider now the contribution of (4.12) to (4.9). Using the identity

$$\frac{\sin k|\vec{x}|}{|\vec{x}|} = \frac{1}{2} \nabla^2 \left[\frac{1 - \cos k|\vec{x}|}{k} \right] - \frac{1}{2} k \cos k|\vec{x}| \ , \tag{4.16}$$

which can itself be obtained from (4.10), we may write

$$\begin{aligned} \Lambda_2 &= -\frac{M^2}{2g^2} \sum_{i \neq j; i,j=1}^{2} \lambda_i \lambda_j \int_{V_{x_i}} d^3\xi \nabla^k_{(\xi)} [\vec{\nabla}_{(\xi)} \times \varpi(\xi - x)]^l \int_{V_{x_j}} d^3\eta \nabla^k_{(\eta)} [\vec{\nabla}_{(\eta)} \times \varpi(\eta - y)]^m \times \\ &\times \frac{(\xi - \eta)^l (\xi - \eta)^m}{|\vec{\xi} - \vec{\eta}|^2} \left\{ \frac{1}{(2\pi)^2} \int_0^\infty dk \frac{1}{\sqrt{k^2 + M^2}} \left[\frac{1}{2} \nabla^2_{(\xi)} \left(\frac{1 - \cos k|\vec{\xi} - \vec{\eta}|}{k^2} \right) + \right. \right. \\ &- \left. \left. \frac{3}{2} \cos k|\vec{\xi} - \vec{\eta}| \right] \right\} \ . \end{aligned} \tag{4.17}$$

Let us observe now that since $\vec{\xi} = \vec{x} + \vec{r}$ and $\vec{\eta} = \vec{y} + \vec{r}\,'$, where $\vec{r}$ and $\vec{r}\,'$ are the integration variables in $d^3\xi$ and $d^3\eta$, we have $|\vec{\xi} - \vec{\eta}| \stackrel{|\vec{x}-\vec{y}|\to\infty}{\longrightarrow} |\vec{x} - \vec{y}|$. As a consequence, we conclude that we can already take the limit in which the argument is large in the term between brackets in (4.17). The last term between brackets will vanish because of the Riemann-Lebesgue lemma. The first one will not because of the singularity at $k = 0$. In Appendix D we show that (4.17) can be put in the form

$$\begin{aligned} \Lambda_2 &= -\frac{M^2}{2g^2} \sum_{i \neq j; i,j=1}^{2} \lambda_i \lambda_j \oint_{S_i} d^2\xi^l \nabla^k_{(\xi)} [\vec{\nabla}_{(\xi)} \times \overline{\omega}(\xi - x)]^{j=3} \oint_{S_j} d^2\eta^l \nabla^k_{(\eta)} [\vec{\nabla}_{(\eta)} \times \overline{\omega}(\eta - y)]^{j=3} \times \\ &\times \frac{1}{2(2\pi)^2} \int_0^\infty dk \frac{1}{\sqrt{k^2 + M^2}} \left[\frac{1 - \cos k|\vec{x} - \vec{y}|}{k^2 |\vec{x} - \vec{y}|} \right] |\vec{x} - \vec{y}| \, . \end{aligned} \tag{4.18}$$

The k-integral can be made as before. The surface integrals are evaluated in Appendix C. The result for Λ_2 is the same as for Λ_1, namely,

$$\Lambda_2 = -\frac{\pi^5}{32} \frac{M}{g^2} |\vec{x} - \vec{y}| \, . \tag{4.19}$$

4.4) The 0-loop result

Collecting the contributions from Λ_1 and Λ_2 to the large distance behavior of the magnetic monopole two-point correlation function, we find

$$\langle \mu(x) \mu^\dagger(y) \rangle \stackrel{|\vec{x}-\vec{y}|\to\infty}{\longrightarrow} \exp\left\{ -\frac{\pi^5}{16} \frac{M}{g^2} |\vec{x} - \vec{y}| \right\} \, . \tag{4.20}$$

From this result, we can infer the value of the mass of the quantum monopole at the tree-level:

$$M^{(0)}_{\text{mon}} = (\frac{\pi^4}{64}) \frac{4\pi M}{g^2} = 1.522 \, \frac{4\pi M}{g^2} \, . \tag{4.21}$$

We see that this result is in agreement with the classical mass of the monopole which is found to be in the range $(1 \leftrightarrow 1.787)\ \frac{4\pi M}{g^2}$ [10, 11].

5) Conclusion

The method of quantization of magnetic monopoles, based on the order disorder duality which exists between the monopole operator and the lagrangian fields, proves to be very convenient because of the fact that the evaluation of monopole operator correlation functions reduces to a standard computation of quantum field theory in the presence of an external field.

Our zero-loop result for the monopole mass falls in the range of values which are obtained for the classical mass [11]. Note however that even at the level of our zero loop computation we are effectively taking into account nontrivial quantum corrections to the monopole correlation function. This follows from the very fact that we are describing the monopole excitations by means of a fully quantized operator.

It would be very interesting to perform the same calculations for the case of a grand-unified model, like $SU(5)$, for instance. Also, the introduction of finite temperature effects would allow us to study the temperature dependence of the magnetic monopole mass, maybe with important cosmological consequences.

It also would be interesting to verify how the monopole correlation functions deviate from the asymptotic large distance regime. In order to do that one should weigh the importance of the graphs with more than two legs which were not considered here. It would be extremely interesting then to study the possible effects of quantum corrections in processes like the monopole catalysis of baryon decay or the monopole production in spontaneous symmetry breaking phase transitions in the early universe.

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A) Two-Legs Graphs

Let us show here that graphs with two legs are the only ones which contribute to the monopole correlation function in the long distance regime. Any two-legs graph can be written in a form as in (4.6):

$$\Lambda^{(2)} = \frac{1}{g^2}\int d^3\xi_\mu \int d^3\eta_\nu\, \overline{\omega}^\alpha(\xi)\overline{\omega}^\beta(\eta)\left[\epsilon^{\mu\alpha\sigma\rho}\partial_\sigma^{(\xi)}\epsilon^{\nu\beta\lambda\gamma}\partial_\lambda^{(\eta)}\right]\left(\vec{\nabla}_{(\xi)}\cdot\vec{\nabla}_{(\eta)}\right)\delta^{\gamma\rho}F(\xi-\eta)\,. \quad \text{(A.1)}$$

Observe that since $\Lambda^{(2)}$ is dimensionless, $F(\xi-\eta)$ must have dimension of $(mass)^2$. Writing (for $\xi^4=\eta^4$)

$$\delta^{\gamma\rho}F(\xi-\eta) = \delta^{\gamma\rho}\int \frac{d^3k}{(2\pi)^4}\frac{e^{i\vec{k}\cdot(\vec{\xi}-\eta)}}{|\vec{k}|^2}C^{(2)}(\vec{k};M)\,, \quad \text{(A.2)}$$

we see that $C^{(2)}$ has dimension of $mass$.

In the large distance regime, we have seen in Sec. 4 that we can write (A.2) in the form

$$\delta^{\gamma\rho}F(\xi-\eta) \overset{|\vec{x}-\vec{y}|\to\infty}{\longrightarrow} [\partial\partial]^{\gamma\rho}C^{(2)}(0;M)f^{(2)}(|\vec{x}-\vec{y}|)\,, \quad \text{(A.3)}$$

where $[\partial\partial]^{\gamma\rho}$ is either $\nabla^\gamma\nabla^\rho$ or $\nabla^2\hat{x}^\gamma\hat{x}^\rho$, for Λ_1 and Λ_2, respectively. This structure remains valid for every two-legs graph at all orders. Only $C^{(2)}(\vec{k};M)$ changes. At 0-loop, we saw that

$$C^{(2)}(\vec{k};M)_0 = \frac{M^2}{\sqrt{|\vec{k}|^2+M^2}}\,. \quad \text{(A.4)}$$

According to what we have seen in Sec. 4 the long distance behavior of $\Lambda^{(2)}$ must be given by

$$\Lambda^{(2)} \sim C^{(2)}(0;M)f(|\vec{x}-\vec{y}|)\,. \quad \text{(A.5)}$$

Since $C^{(2)}(0;M)$ has dimension of $mass$ ($C^{(2)}(0;M)_0 = M$) it follows that for any two-legs graph $f(|\vec{x}-\vec{y}|)\sim|\vec{x}-\vec{y}|$ at large distances. This behavior leads to the exponential decay of the monopole correlation function.

Let us consider now a generic 4-legs graph. This has the form

$$\Lambda^{(4)} = \frac{1}{g^4}\int\prod_{i=1}^{4} d^3\xi_i^{\mu_i}\ \overline{\varpi}^{\alpha_i}(\xi_i) \sum_{combinations} \left\{\left[\epsilon^{\mu_1\alpha_1\sigma_1\rho_1}\partial_{\sigma_1}^{(\xi_1)}\right]\dots\left[\epsilon^{\mu_4\alpha_4\sigma_4\rho_4}\partial_{\sigma_4}^{(\xi_4)}\right]\times \right.$$
$$\left. \times\ (\nabla_{\xi_1}\cdot\nabla_{\xi_2})(\nabla_{\xi_3}\cdot\nabla_{\xi_4})\delta^{\rho_1\rho_2}\delta^{\rho_3\rho_4}\right\} F(\xi_1,\dots,\xi_4)\ , \tag{A.6}$$

where $F(\xi_1,\dots,\xi_4)$, the analog of (A.2), is given by

$$F(\xi_1,\dots,\xi_4) = \int\frac{d^3k_1}{(2\pi)^4}\cdots\frac{d^3k_3}{(2\pi)^4}\frac{e^{i\vec{k}_1\cdot(\vec{\xi}_1-\vec{\xi}_4)}e^{i\vec{k}_2\cdot(\vec{\xi}_2-\xi_4)}e^{i\vec{k}_3\cdot(\xi_3-\xi_4)}}{|\vec{k}_3|^2(-\vec{k}_1\cdot\vec{k}_2)}C^{(4)}(\vec{k}_i;M)\ . \tag{A.7}$$

Observe that $F(\xi_1,\dots,\xi_4)$ has dimension of $(mass)^4$ and $C^{(4)}(\vec{k}_i;M)$ of $(mass)^{-1}$.

As in (4.5), for large distances, we can write

$$\delta^{\rho_1\rho_2}\delta^{\rho_3\rho_4}F(\xi_1,\dots,\xi_4) = [\partial\partial\partial\partial]^{\rho_1\rho_2\rho_3\rho_4}C^{(4)}(0;M)f^{(4)}(|\vec{x}-\vec{y}|)\ , \tag{A.8}$$

where $[\partial\partial\partial\partial]^{\rho_1\rho_2\rho_3\rho_4}$ is the obvious generalization of $[\partial\partial]^{\rho_1\rho_2}$ containing four derivatives. Following the same procedure as in the case of the two-legs graphs we conclude that the large distance behavior of a four-legs graph is given by

$$\Lambda^{(4)}(x-y) \stackrel{|\vec{x}-\vec{y}|\to\infty}{\longrightarrow} C^{(4)}(0;M)f^{(4)}(|\vec{x}-\vec{y}|)\ . \tag{A.9}$$

Since $C^{(4)}$ has dimension of $(mass)^{-1}$ we conclude that $f^{(4)} \stackrel{|\vec{x}-\vec{y}|\to\infty}{\longrightarrow} |\vec{x}-\vec{y}|^{-1}$ and therefore at large distances,

$$\Lambda^{(4)}(x-y) \sim \frac{1}{|\vec{x}-\vec{y}|}\ . \tag{A.10}$$

We can easily perform the same analysis in the case of a graph with 2n-legs. We are led, then, to the conclusion that in general

$$\Lambda^{(2n)}(x-y) \stackrel{|\vec{x}-\vec{y}|\to\infty}{\longrightarrow} [\mathcal{M}|\vec{x}-\vec{y}|]^{3-2n} \tag{A.11}$$

where $\mathcal{M}$ has dimension of *mass*. As a consequence, we see that only two-legs graphs contribute to the asymptotic large distance behavior of the monopole correlation function.

B) The Equation (4.6)

Let us show here that the last term in (4.6) does indeed vanish. Integrating by parts, we have an expression proportional to

$$\int d^4z d^4z' \partial^\alpha \tilde{\mathcal{A}}^\alpha_\mu(z) \partial'^\beta \tilde{\mathcal{A}}^\beta_\nu(z') \delta^{\mu\nu} F(z-z') \, . \tag{B.1}$$

We can now write

$$\begin{aligned} \partial^\alpha \tilde{\mathcal{A}}^\alpha_\mu(z) &= -\frac{1}{g}\partial^{(z)}_\alpha \int_{V_x} d^3\xi_\mu \, \varpi^\alpha(\xi - x)\delta^4(z-\xi) = \\ &= -\frac{1}{g}\int_{V_x} d^3\xi_\mu \, \varpi^\alpha(\xi - x)\partial^{(z)}_\alpha \delta^4(z-\xi) - \frac{1}{g}\hat{n}^\mu \oint_{S_x} d^2\xi_\alpha \, \varpi^\alpha(\xi-x)\delta^4(z-\xi) \, , \end{aligned} \tag{B.2}$$

where the last term comes from S_x, the boundary of the volume V_x ($\hat{n}$ is the unit vector in the $d^3\xi^\mu$ direction. Using Gauss' theorem in the last term of (B.2) we see that we have, for an arbitrary function $J^\mu(z)$

$$\begin{aligned} \int d^4z \partial^\alpha \tilde{\mathcal{A}}^\alpha_\mu(z) J^\mu(z) &= -\frac{1}{g}\int d^3\xi_\mu \, \varpi^\alpha(\xi-x)(-)\partial^{(\xi)}_\alpha J_\mu(\xi) - \frac{1}{g}\int d^3\xi^\mu \, \partial^{(\xi)}_\alpha \left[\varpi^\alpha(\xi-x)J_\mu(\xi)\right] = \\ &= -\frac{1}{g}\int d^3\xi_\mu \left[\partial^{(\xi)}_\alpha \, \varpi^\alpha(\xi - x)\right] J_\mu(\xi) \, . \end{aligned} \tag{B.3}$$

This expression vanishes with the divergence of ϖ^α.

C) Evaluation of the Surface Integrals

Let us compute here the surface integrals appearing in (4.13) and (4.18). Let us call I_1 the term involving the surface integrals appearing in (4.13):

$$I_1 = \oint_{S_x} d^2\xi^l \nabla^k_{(\xi)} (\vec{\nabla} \times \varpi(\vec{\xi} - \vec{x}))^l \oint_{S_y} d^2\eta^m \nabla^k_{(\eta)} (\vec{\nabla} \times \varpi(\vec{\eta} - \vec{y}))^m \, . \tag{C.1}$$

In spherical coordinates we have that $\varpi = (-\theta \sin\varphi, \theta\cos\varphi, 0) = \theta\hat{\varphi}$. The rotational of ϖ is given by

$$\vec{\nabla} \times \overline{\omega} = \hat{r}\, \frac{1 + \theta \cot\theta}{r} - \hat{\theta}\, \frac{\theta}{r}\,. \tag{C.2}$$

In (C.1), $d^2\xi^l$ and $d^2\eta^m$ are the elements of area of the spherical surfaces S_x and S_y, respectively, expressed as $d^2\xi^l \to r^2 d\varphi d\theta \sin\theta\, \hat{r}$ and $d^2\eta^m \to r'^2 d\varphi' d\theta' \sin\theta'\, \hat{r}'$. From the integration by parts done in (4.13), we see that it is the radial term of $\vec{\nabla} \times \overline{\omega}$ that is going to contribute in (C.1). We have then that

$$\hat{r}^l \nabla^k (\vec{\nabla} \times \overline{\omega})^l_{\text{radial}} = -\frac{\hat{r}^k}{r^2}(1 + \theta\cot\theta) + \frac{\hat{\theta}^k}{r^2}\left(\cot\theta - \frac{\theta}{\sin^2\theta}\right)\,. \tag{C.3}$$

Therefore, we get for the surface integral

$$\oint_S dS \hat{r}^l \nabla^k (\vec{\nabla}\times\overline{\omega})^l_{\text{radial}} = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta \left[-\hat{r}^k(1 + \theta\cot\theta) + \hat{\theta}^k\left(\cot\theta - \frac{\theta}{\sin^2\theta}\right)\right]. \tag{C.4}$$

Substituting the vectors $\hat{r} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ and $\hat{\theta} = (\cos\theta\cos\varphi, \cos\theta\sin\varphi, -\sin\theta)$ in (C.4) and performing the angular integrations, one obtains

$$\oint_S dS\, \hat{r}^l \nabla^k (\vec{\nabla} \times \overline{\omega})^l_{\text{radial}} = \frac{\pi^3}{2}(0, 0, 1)^k\,. \tag{C.5}$$

Thus, we get for I_1, Eq. (C.1), the result

$$I_1 = \frac{\pi^6}{4}\,. \tag{C.6}$$

Let us call I_2 the term involving the surface integrals in (4.18):

$$I_2 = \oint_{S_x} d^2\xi^l \nabla^k_{(\xi)} (\vec{\nabla} \times \overline{\omega}(\vec{\xi}))^{j=3} \oint_{S_y} d^2\eta^l \nabla^k_{(\eta)} (\vec{\nabla} \times \overline{\omega}(\vec{\eta}))^{j=3}\,, \tag{C.7}$$

where, we used the fact that $|\vec{\xi} - \vec{\eta}| \overset{|\vec{x}-\vec{y}|\to\infty}{\longrightarrow} |\vec{x} - \vec{y}|$ and, without loss of generality, chose $(\vec{x} - \vec{y})/|\vec{x} - \vec{y}|$ in the 3-direction. From (C.2), $(\vec{\nabla} \times \overline{\omega})^{j=3}$ is given by

$$(\vec{\nabla} \times \overline{\omega})^{j=3} = \frac{\theta + \sin\theta\cos\theta}{r\sin\theta}\,. \tag{C.8}$$

From (C.8) we have that

$$\nabla^k (\vec{\nabla} \times \varpi)^{j=3} = -\frac{\hat{r}^k}{r^2} \frac{\theta + \sin\theta\cos\theta}{\sin\theta} + \frac{\hat{\theta}^k}{r^2} \frac{\sin\theta\cos^2\theta - \theta\cos\theta}{\sin^2\theta} . \tag{C.9}$$

From (C.7) and (C.9) we get that

$$\oint_S dS\, \hat{r}^l \nabla^k (\vec{\nabla} \times \varpi)^{j=3} = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \left[-\hat{r}^l \hat{r}^k \left(\theta + \sin\theta\cos\theta\right) + \hat{r}^l \hat{\theta}^k \frac{\sin\theta\cos^2\theta - \theta\cos\theta}{\sin\theta} \right] . \tag{C.10}$$

Substituting the vectors $\hat{r}$ and $\hat{\theta}$ in (C.10) and performing the angular integrations, we obtain the double-vector:

$$\oint_S dS\, \hat{r}^l \nabla^k (\vec{\nabla} \times \varpi)^{j=3} = -\frac{\pi^3}{2} \left((1,0,0)^k, (0,1,0)^k, (0,0,0)^k \right)^l . \tag{C.11}$$

I_2, Eq. (C.7), is given by taking the scalar product of (C.11) with itself:

$$I_2 = \frac{\pi^6}{2} . \tag{C.12}$$

D) The Equation (4.18)

Let demostrate here that Eq. (4.17) can be put in the form (4.18). Writing $\nabla^2_{(\xi)} = -\vec{\nabla}_{(\xi)} \cdot \vec{\nabla}_{(\eta)}$ we see that (4.17) is of the form

$$\begin{aligned} &\int d^3\xi d^3\eta f(\xi) g(\eta) \vec{\nabla}_{(\xi)} \cdot \vec{\nabla}_{(\eta)} F(\xi - \eta) = \\ &\quad = \int d^3\xi d^3\eta \left(\vec{\nabla}_{(\xi)} \cdot \vec{\nabla}_{(\eta)} \right) \left[f(\xi) g(\eta) F(\xi - \eta) \right] + 3 \text{ additional terms} . \end{aligned} \tag{D.1}$$

The three aditional terms in the above expression always involve an integral of the type

$$\int d^3\eta T^{jkl}(\eta) \int d^3\xi \partial^l_{(\xi)} \partial^k_{(\xi)} \left[\vec{\nabla} \times \varpi(\xi - x) \right]^i \partial^i_{(\xi)} |\xi - \eta| F^j(\xi - \eta) , \tag{D.2}$$

where we used the identity $\frac{x^i}{|\vec{x}|} = \partial^i |\vec{x}|$. Because of the presence of the rotational, we can make the ∂^i derivative total and apply Gauss' theorem to write the last integral as

$$\int d^3\eta T^{jkl}(\eta) \oint d^2\xi^i \partial^l_{(\xi)} \partial^k_{(\xi)} \left[\vec{\nabla} \times \overline{\omega}(\xi - x)\right]^i \left[|x-y| F^j(x-y)\right] , \tag{D.3}$$

where we used the fact that $|\xi - \eta| \stackrel{|\vec{x}-\vec{y}|\to\infty}{\longrightarrow} |x-y|$. Making $\partial^l_{(\xi)} = -\partial^l_{(x)}$ when acting on $\overline{\omega}(\xi - x)$ we immediately see that the above expression is proportional to

$$-\partial^l_{(x)} \oint_{S_i} d^2\xi_i \partial^k_{(\xi)} \left[\vec{\nabla} \times \overline{\omega}(\xi - x)\right]^i . \tag{D.4}$$

In Appendix C, Eq. (C.5), we showed that the above integral is a pure number and therefore independent of x. As a consequence, the above derivative vanishes and all the three aditional terms in (D.1) are equal to zero. Using the theorem of Gauss in the first term on the r.h.s. of (D.1) immediately leads to (4.18). It is interesting to note that the vanishing of the three terms which are not total derivatives in (D.1) depends crucially on the presence of the factors $\partial^i|\xi - \eta| = (\xi - \eta)^i/|\xi - \eta|$. These terms would no longer vanish, for instance, if we tried to apply the same procedure of this Appendix to (4.9).

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Figure Captions

Figure 1: Volume configuration in the definition of the external field $\tilde{\mathcal{A}}^a_\mu(z;x)$;

Figure 2: Vertices involving the external field $\tilde{\mathcal{A}}^a_\mu$ (curly line) contributing at zero loop. Vertices with dark dots represent derivative vertices (gauge dependent). ($a = 1,\, 2$);

Figure 3: Graphs contributing to the asymptotic behavior of $\Lambda(x-y)$, in Eq. (4.1), at zero loop. In the symmetric phase the contribution to Λ is given only by the first two graphs;

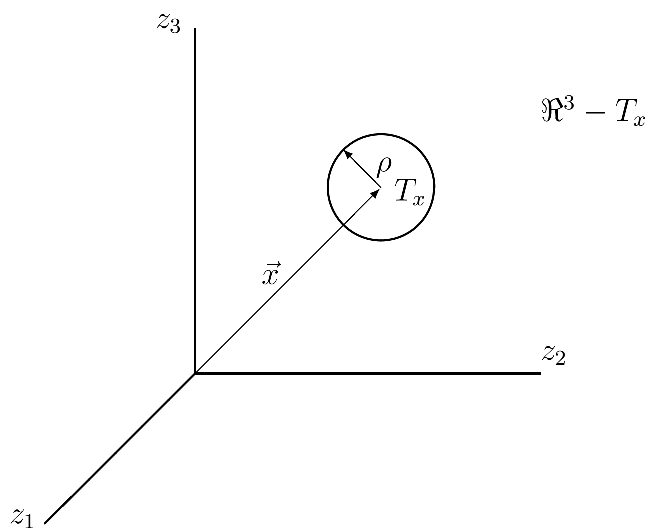
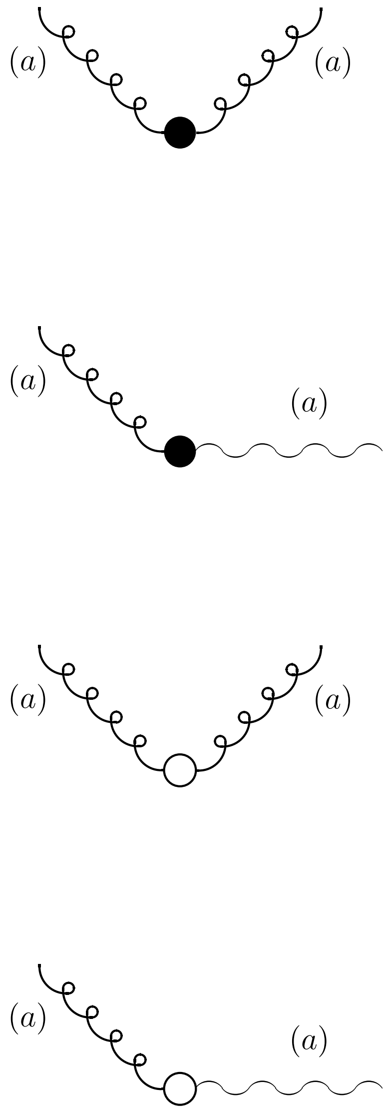


Figure 1



$$-\frac{\xi}{2}(\partial_\mu \tilde{\mathcal{A}}^\mu_a)^2$$

$$\xi \partial_\mu \tilde{\mathcal{A}}^\mu_a \partial_\nu W^\nu_a$$

$$-\frac{M^2}{2}(\tilde{\mathcal{A}}^\mu_a)^2$$

$$M^2 \tilde{\mathcal{A}}^\mu_a W^\mu_a$$

Figure 2

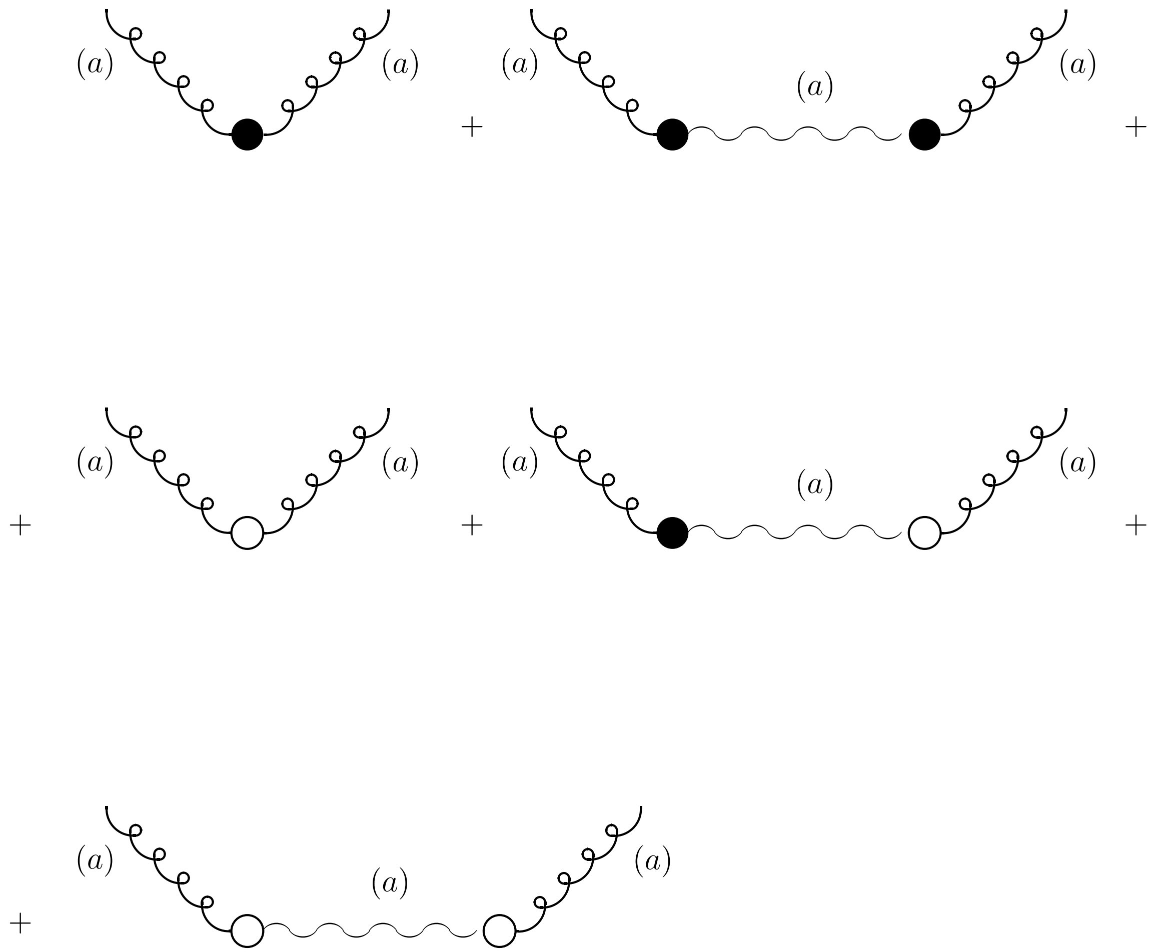


Figure 3