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Effects of the sound speed of quintessence on the microwave background and large scale structure

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We consider how quintessence models in which the sound speed differs from the speed of light and varies with time affect the cosmic microwave background and the fluctuation power spectrum. Significant modifications occur on length scales related to the Hubble radius during epochs in which the sound speed is near zero and the quintessence contributes a non-negligible fraction of the total energy density. For the microwave background, we find that the usual enhancement of the lowest multipole moments by the integrated Sachs-Wolfe effect can be modified, resulting in suppression or bumps instead. Also, the sound speed can produce oscillations and other effects at wave numbers $k > 10^{-2}$ h/Mpc in the fluctuation power spectrum.

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One of the greatest challenges in cosmology today is to identify the nature of the dark energy component that comprises most of the energy density of the universe and that is causing the expansion of the universe to accelerate [1]. Two candidates are a cosmological constant (or vacuum density) and quintessence [2], a dynamical energy component with negative pressure. Distinguishing the two is important for cosmology in order to refine our knowledge of the composition of the universe and to trace more accurately its evolution. It is even more important for fundamental physics since it informs us how we must modify unified theories to incorporate dark energy.

One way to distinguish whether the dark energy is due to a cosmological constant or quintessence is to measure the equation of state, w , the ratio of the pressure p to the energy density ρ . A cosmological constant always has $w = -1$ whereas a scalar field generally has a $w(z)$ that differs from unity and varies with redshift z . Through measurements of supernovae, large-scale structure and the cosmic microwave background (CMB) anisotropy, the equation of state may be determined accurately enough in the next few years to find out whether or not w is different from -1 .

A second way to distinguish the nature of dark energy is to measure its sound speed to determine if it is different from unity. The sound speed can be detected because it affects the perturbations in the quintessence energy distribution. This approach is less generic because the sound speed in many models of quintessence in the literature is equal to unity (the speed of light), e.g., models in which quintessence consists of a scalar field (ϕ) with canonical kinetic energy density ($X \equiv \frac{1}{2}(\partial_\mu \phi)^2$) and a positive potential energy density ($V(\phi)$). However, in general, the sound speed can differ from unity and vary with time. Detecting these effects is an independent way of showing that dark energy does not consist of a cosmological constant.

An important motivating example is k essence [3]. In these models, the k essence undergoes two transitions in its

behavior, one beginning at the onset of matter domination and a second when k essence overtakes the matter density. During the radiation-dominated era, the k -essence energy tracks the radiation, falling as $1/a^4$ where a is the scale factor. This tracking feature is significant because it explains how dark energy can remain a subdominant but not completely negligible component prior to matter-radiation equality. Generic models with this behavior (k essence is only one example) are known as “trackers” [4]. The tracking property is relevant to our purpose here because the sound speed can have greater influence if the dark energy is non-negligible for a broad range of red shift. (A cosmological constant becomes completely negligible by redshift $z=2$, but this is not the case for typical tracker-type models.)

What is special about k -essence models is that the onset of the matter-dominated era automatically triggers a change in the behavior of k essence such that it begins to act as an energy component with $w(z) \approx -1$. The effect is achieved with a scalar field by introducing a kinetic energy density which contains higher order derivative terms (that is, terms non-linear in X) and which results in attractor solutions with the desired behavior. This kind of model is appealing because it explains dynamically why cosmic acceleration only begins well after matter domination, just as we observe, which is one of the puzzling aspects of dark energy. When k essence overtakes the matter density, $w(z)$ changes to another value greater than -1 , the precise value of which depends on the detailed model. The way in which the attractor behavior of k essence is automatically controlled by the background equation of state is important motivation for cosmologists to consider k -essence models, specifically. However, the analysis in this paper is more general.

For the purposes of this paper, what is significant about the k -essence example is an indirect consequence of the tracker behavior: the sound speed c_s undergoes dramatic changes as $w(z)$ passes through a series of transitions in evolving from the radiation- to the matter -to the k -essence-

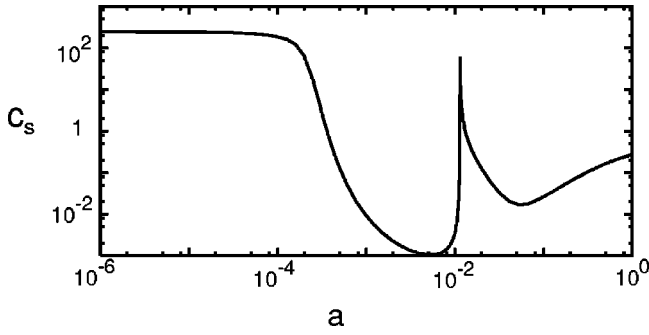


FIG. 1. The speed of sound as a function of the scale factor a ($a_{\text{today}}=1$) for a k -essence model studied in Ref. [3]. Note the changes in c_s between the last scattering surface and the present epoch triggered by the transformations in the background equation of state.

dominated epochs [5]. The behavior of c_s in a particular k -essence model [3] is shown in Fig. 1. Here the sound speed (i) is relativistic in the radiation-dominated epoch, (ii) has rapid changes between zero and relativistic as $w(z)$ settles towards the attractor solution, which corresponds to $c_s^2 \rightarrow 0$ and $w(z) \rightarrow -1$, and, then, (iii) increases somewhat at recent redshift when k essence overtakes the matter density. The equation of state in tracker models must always have these three phases. Without knowing what the dark energy consists of, it is reasonable to imagine a generic class of dark energy models where the sound speed during each phase varies in a manner similar to the specific model described here. For another example, see Ref. [6]. In particular, changes in sound speed are related to the changes in the scalar field as it moves rapidly between the radiation tracking and attractor solution in phase (ii), and asymptotes to the $w = -1$ attractor point (iii).

Our study here does not rely specifically on non-linear kinetic energy or other aspects specific to k essence, but simply on the equation of state $w(z)$ and the sound speed $c_s(z)$ as a function of redshift z . The purpose of this paper is to study possible effects of the sound speed and its time variation on the cosmic microwave background anisotropy and on large scale structure. In an earlier paper [7], the subtle changes due to sound speed in the position and shape of the acoustic peaks in the CMB power spectrum were investigated and it was shown that the effect of the sound speed could be discriminated from other physical effects provided that the k -essence density is non-negligible (greater than 1%) at last scattering. Here we focus on the large angular scales corresponding to the quadrupole and first few higher multipole moments. We show that $c_s^2 \ll 1$ at small redshift can produce a suppression of these first few multipole moments, an effect similar to the low quadrupole observed in the Cosmic Background Explorer (COBE) data [8,9]. We also show how rapid variations in the sound speed, as in the k -essence example, can produce bumps, oscillations and other novel features at small scales in the mass power spectrum. While the precise effect depends on the detailed model, we examine a range of models to determine the gross consequences of a changing sound speed. As in [7], we restrict our attention to flat universes only.

As noted above, our calculations and examples do not assume that the dark energy consists of k essence specifically, which is a purely kinetic action for the scalar field. In fact, our computer code, a modification of the standard CMBFAST program [2,10], effectively treats the quintessence component the same as a scalar field with a potential. The code automatically determines a potential that produces the desired $w(z)$. Independently, the perturbation equations are modified to include the desired sound speed, $c_s(z)$.

The effect of the speed of sound on the CMB perturbation equations is such that for $c_s^2 \ll 1$, k -essence fluctuations are enhanced via gravitational instability by the cold dark matter (CDM) potentials. For those familiar with the code, the modifications are straightforward. The perturbed line element is

$$ds^2 = a^2(\tau)[d\tau^2 - (\delta_{ij} + h_{ij})dx^i dx^j] \quad (1)$$

where δ_{ij} is the unperturbed spatial metric, and h_{ij} is the metric perturbation. We shall use h to represent the trace of the spatial metric perturbation. The effect we are examining is due to adiabatic only perturbations to the k -essence stress-energy in the synchronous gauge for a mode with wave number k (we omit the k subscripts here):

$$\begin{aligned} \delta\rho &= -2\rho \frac{\delta\phi}{\phi} - (\rho + p) \frac{\delta y}{y} c_s^{-2} \\ \delta p &= -2p \frac{\delta\phi}{\phi} - (\rho + p) \frac{\delta y}{y} \\ \theta &= \frac{1}{\sqrt{2}} k^2 y \delta\phi \end{aligned} \quad (2)$$

where $y \equiv 1/\sqrt{X}$ and θ is the divergence of the fluid velocity. The density contrast, $\delta \equiv \delta\rho/\rho$, obeys the equation

$$\dot{\delta} = -(1+w) \left(\theta + \frac{1}{2} \dot{h} \right) - 3\mathcal{H} \left(\frac{\delta p}{\delta\rho} - w \right) \delta, \quad (3)$$

where the derivative is with respect to conformal time and $\mathcal{H} \equiv \dot{a}/a$. The relation between δp and $\delta\rho$, derived using Eqs. (2) above combined with the background equation of motion, is

$$\delta p = c_s^2 \delta\rho + \frac{\theta\rho}{k^2} [3\mathcal{H}(1+w)(c_s^2 - w) + \dot{w}], \quad (4)$$

which leads to a simplified evolution equation for the velocity gradient

$$\dot{\theta} = (3c_s^2 - 1)\mathcal{H}\theta + c_s^2 k^2 \delta / (1+w). \quad (5)$$

We can see in Eq. (5) that a small sound speed will cause the velocity gradient to decay; with the conventional gauge choice, $\theta_{\text{cdm}} = 0$, the inhomogeneities in the k essence will describe a fluid which is comoving with the cold dark matter. From Eq. (4), we see that the second term on the right-hand side (RHS) will be negligible even on scales approaching the

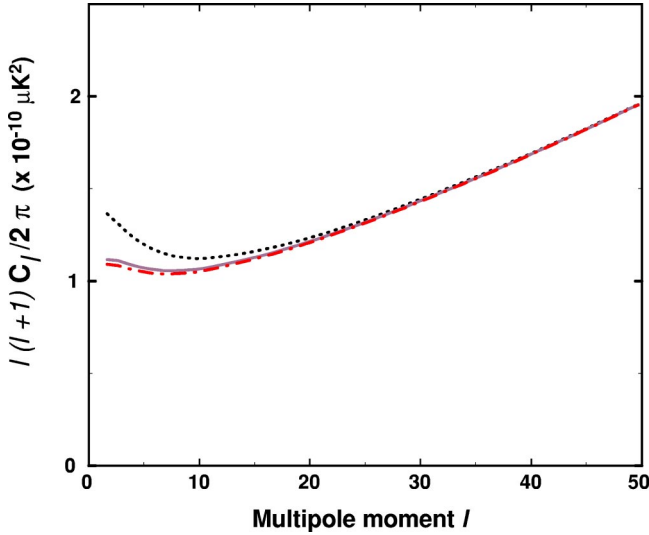


FIG. 2. Comparison of the lowest multipole moments of the CMB temperature power spectrum for a series of models with $w = -0.8$: (a) $c_s = 1$ (dotted); (b) $c_s = 1$ until $z = 5$ and then $c_s = 0$ for $z < 5$ (solid); and (c) $c_s = 0$ for all z (dot-dashed).

horizon. The overall effect is that the pressure fluctuations δp are weak and k -essence fluctuations are enhanced via gravitational instability by the cold dark matter (CDM) gravitational potentials. The CMBFAST code takes $w(a)$ and $c_s(a)$ as inputs, so it is possible to manually adjust these functions to have any values (including, of course, $c_s = 1$).

As a first set of cases we consider a sequence of models in which the equation of state is constant, $w = -0.8$, and we vary the sound speed. Note that this example does not correspond to tracker-type evolution and, like the cosmological constant, the quintessence component becomes negligible at $z > 2$ or so. Figure 2 compares three cases: (a) $c_s = 1$ (dotted); (b) $c_s = 1$ for $z \geq 5$ and then $c_s = 0$ for $z < 5$; and (c) $c_s = 0$ for all z . All models have $\Omega_m = 0.25$, $H_0 = 70$ km/s/Mpc, and $\Omega_b = 0.04$, where $\Omega_{m(b)}$ is the ratio of the matter (baryon) density to the critical density and H_0 is the Hubble parameter.

We observe the following features:

(1) The (late-time) integrated Sachs-Wolfe (ISW) effect is suppressed if the sound speed is near zero at recent red shift. The ISW effect occurs in models with $\Omega_m < 1$ because the gravitational potential decreases as photons propagate between the last scattering surface and today. The net effect is an increase in the multipoles on angular scales which enter the horizon when $\Omega_m < 1$, that is to say, the low- ℓ multipole moments. This is the effect predicted for models with a cosmological constant, for example. Dark energy with sound speed near zero suppresses this effect because the fluctuations in the quintessence component add to the gravitational potential and compensate for the decrease one would obtain if the quintessence were smoothly distributed [11].

(2) For constant w models the ISW suppression is only weakly dependent on the time dependence of the sound speed. For example, the lower two curves in Fig. 2 nearly overlap. The reason for this is apparent—the quintessence component is a negligible fraction of the energy density for

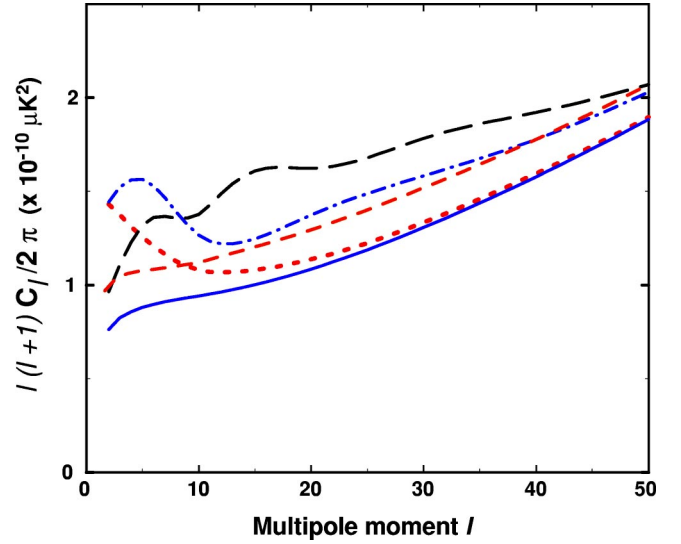


FIG. 3. Comparison of the lowest multipole moments of the CMB temperature power spectrum for a series of models with the same $w(z)$ (in this case, corresponding to the k -essence model in Fig. 1 and Ref. [3]) but different $c_s(z)$: (a) $c_s = 1$ (dotted); (b) $c_s = 1$ for $z > 10$ and $c_s = 0$ for $z < 10$ (solid); the k -essence model shown in Fig. 1 (dot-dashed); and two variations (short- and long-dashed) with rapid variations in c_s^2 (spikes) as described in the text.

$z > 2$, so the behavior of the sound speed is only important over a narrow range of recent redshift.

We have not shown a plot of the multipole moments for large values of ℓ because the curves are essentially degenerate over this range. Similarly, the mass power spectra for these models are virtually identical. Hence, the only effect of the sound speed for these non-tracker type models with nearly constant w is a modest suppression of the low- ℓ multipole moments in the CMB.

In Fig. 3, we show the small- ℓ CMB multipole moments for models with the same $w(z)$, where w is now z dependent. As a specific example, we choose $w(z)$ in these examples to correspond to the k -essence model shown in Fig. 1 and discussed in Ref. [3]. We choose a range of illustrative $c_s(z)$ behaviors, whose other consequences we will return to examine in Figs. 4 and 5. As before, all models have current values of $\Omega_m = 0.25$, $H_0 = 70$ km/s/Mpc, and $\Omega_b = 0.04$. The details of $w(z)$ are not important here except that it has the feature that the dark energy is a tracker component that is non-negligible at last scattering and earlier, roughly 10% of the critical density. We note the following features:

(1) Comparing the models with $c_s = 1$ (dotted) and with the case with $c_s = 1$ for $z > 10$ but $c_s = 0$ for $z < 10$ (solid), there is greater suppression than in Fig. 2 because the quintessence component is non-negligible for a broader range of red shift, enhancing its clustering effect. (We have considered this example of time-variation because it is generally plausible that the sound speed of the quintessence component changes as it overtakes the matter density, as occurs in the case of k essence.)

(2) More general time-variation of the sound speed can produce bumps and wiggles in the low- ℓ multipole moments. The remaining examples in Fig. 3 are cases where the

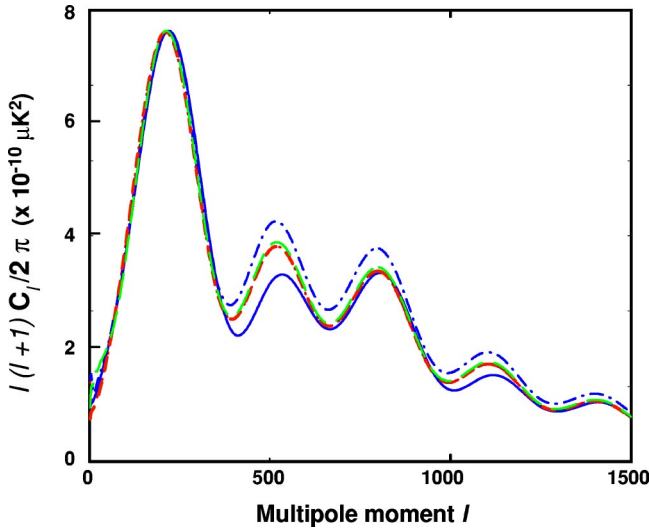


FIG. 4. Comparison of higher multipole moments of the CMB temperature power spectrum for the same sequence of models as in Fig. 3. The spectra have been normalized so that the amplitudes match at the top of the first acoustic peak.

sound speed corresponds to the actual k -essence model shown in Fig. 1 (dot-dashed), or simple variations. For the variations, we have considered cases where the sound speed is $c_s = 1$ until last scattering, then drops to $c_s = 0$ except for a spike near $z = 100$ where the quintessence component approaches the attractor solution with $w = -1$. While this roughly mimics features that are characteristic of k -essence, here we have bounded $c_s \leq 1$ and treated the spike as a smoothly varying function to illustrate that the unusual features seen in the actual k essence example do not depend on having a sound speed greater than unity or with near-discontinuous behavior. We find that the precise form of the low- ℓ power spectrum is sensitive to the redshift and width

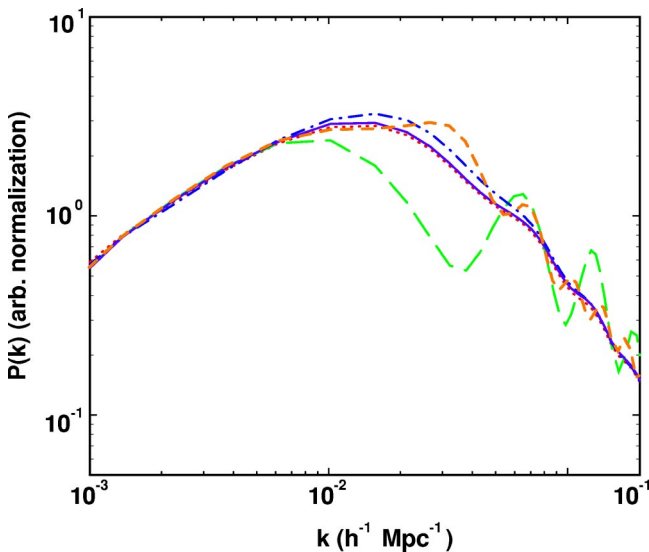


FIG. 5. Comparison of the *shape* of the total fluctuation power spectrum, $P(k)$, as a function of wave number k for the same sequence of models as in Fig. 3. The normalization of the curves is arbitrary.

of the spike, producing in some cases bumps and wiggles in the small- ℓ multipole moments.

Curiously, the COBE data suggest a suppression, and perhaps even a bump-like feature in the low- ℓ multipoles [7,8]. The data are not decisive because the cosmic variance and experimental error are large for the low- ℓ multipoles. Forthcoming data from the MAP and Planck satellite experiments may lead to some improvement.

Figure 4 shows the full CMB power spectra for the same sequence of models as in Fig. 3. Recall that these models are completely identical except for differences in $c_s^2(z)$. These spectra are roughly similar, but could be easily distinguished in near future CMB power spectrum measurements, e.g., by the Microwave Anisotropy Probe (MAP) and Planck satellites. However, detecting and distinguishing $c_s^2(z)$ is not as simple as the Fig. 4 may suggest. The modest differences in peak heights and positions seen here can be approximately mimicked by varying other parameters. So, the challenge becomes to separate effects of $c_s^2(z)$ from those of the other parameters. As shown in Ref. [3], the effects are not completely degenerate, so highly precise measurements of the power spectra can make the separation, in principle. However, the differences are small enough that a systematic search through possible forms for $c_s^2(z)$ along with the standard parameters is required in most cases to detect them, and this is not part of any current fitting procedure. Observing anomalies in the low- ℓ multipole moments and in the mass power spectrum (see below) as discussed in this paper would provide strong motivation for developing such a procedure.

We note that similar suppression of the low- ℓ multipoles has been obtained previously by keeping $c_s^2 = 1$ but varying the equation of state $w(z)$ [2]. The greater is the value of w , the greater are the fluctuations in the quintessence field, which is the same trend one obtains by lowering the sound speed. However, if $c_s^2 = 1$, the suppression of multipoles only occurs if $w(z)$ is greater than ~ -0.2 today. This is inconsistent with the observed acceleration of the universe, which requires $w(z) < -1/3$ [1,12]. On the other hand, keeping $w(z) \approx -1$ today and varying the sound speed history is an interesting alternative because it is physically motivated and does not run into conflict with any other current observations.

Similarly, one could modify the low- ℓ multipoles by altering the primordial power spectrum, for example, by introducing an inflation potential with bumps or dips strategically placed so as to produce suppression or bumps in the low- ℓ multipoles. This solution is unappealing. Introducing such features at any point in the inflaton potential requires very awkward fine-tuning; but, in this case, where we are seeking to alter only the low- ℓ multipoles, such an approach is even more awkward because the features in the inflaton potential must be introduced in a way so as to affect specifically those wavelengths that recently entered the horizon (rather than longer or shorter wavelengths). There is simply no good reason for this. In contrast, it is natural to associate the suppression or bumps in the low- ℓ multipoles with dark energy because it only dominates at late times and so will naturally affect modes whose wavelengths only recently entered the horizon.

Figure 5 compares the total fluctuation power spectrum (including contributions from fluctuating components) for the same sequence of models as in Fig. 3. We have intentionally included an example with exaggerated features to illustrate that the sound speed effect can be quite large (in this case, by making an optimal choice of the red shift and width of the spike in c_s). The main effect to be observed here is that there can be enhancement in power, suppression on some scales, oscillations and other features in $P(k)$ at large $k > 0.01$ h/Mpc. Clearly, uncertainty in the sound speed can lead to non-negligible uncertainty in σ_8 and, more generally, the shape of the power spectrum even when all other parameters are fixed. The effect could be important in explaining anomalies that may arise in $P(k)$ as high quality redshift survey data are obtained and compared to CMB measurements.

When $c_s^2 = 0$, the quintessence fluctuations are enhanced, producing the effects in the microwave background and mass power spectrum that we have observed, but the fluctuations do not cluster like matter because the equation of state, w , is different. For example, consider a universe containing only quintessence with $c_s^2 = 0$ and adiabatic perturbations $\delta = \delta\rho/\rho$, where ρ is the quintessence energy density. Since $w \neq 0$, the Jean's instability equation is modified,

$$a^2 \frac{d^2 \delta}{da^2} + \frac{3}{2} a A[c_s^2, w] \frac{d\delta}{da} + \left(\frac{k^2 c_s^2}{\mathcal{H}^2} - \frac{3}{2} B[c_s^2, w] \right) \delta = 0 \quad (6)$$

where a is the Friedmann-Robertson-Walker scale factor and $A[c_s^2, w] = 1 - 5w + 2c_s^2$ and $B[c_s^2, w] = 1 - 6c_s^2 + 8w - 3w^2$ [13]. For $c_s^2 = w = 0$, we have $A[c_s^2, w] = B[c_s^2, w] = 1$ and we obtain the conventional dust-dominated growing

solution, $\delta \propto a$. However, in general, the “growing” solution is $\delta \propto a^\gamma$ where $\gamma = \frac{1}{2}(1 - A + [(1 - A)^2 - 4B])$, which is less than zero for $w < -0.12$ (for all $c_s^2 \geq 0$). Hence, despite the fact that $c_s^2 = 0$, the accelerated expansion dominates and causes the fluctuations to decay. In a universe with a mixture of matter and quintessence, the matter slows the expansion, so the quintessence may cluster *when it is subdominant*. Indeed, it is the gravitational influence of matter fluctuations on quintessence that is enhancing the perturbations in quintessence that have been studied here. However, this clustering is halted once quintessence begins to dominate and the effective value of w becomes negative. The only significant effects are the kinds of changes in the microwave background and mass power spectra seen here.

What we conclude, then, is that modest changes in $c_s(z)$, as motivated by some existing models of dark energy, can produce modest but measurable changes in the CMB and mass power spectra. The highly precise data obtained from the MAP and Planck satellites and from the Sloan Digital Sky Survey may reveal these subtle effects. The precise behavior of the sound speed is, by itself, of limited interest. But, what is important is that the detection of any deviation from $c_s^2 = 1$ would be a direct sign that the dark energy is a complex, dynamical fluid rather than an inert cosmological constant. Hence, it is a target well worth pursuing.

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