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# A First Principles Warm Inflation Model that Solves the Cosmological Horizon/Flatness Problems

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A quantum field theory warm inflation model is presented that solves the horizon/flatness problems. The model obtains, from the elementary dynamics of particle physics, cosmological scale factor trajectories that begin in a radiation dominated regime, enter an inflationary regime and then smoothly exit back into a radiation dominated regime, with nonnegligible radiation throughout the evolution.

PACS number(s): 98.80 Cq, 05.70.Ln, 11.10.Wx

In Press Physical Review Letters 1999

hep-ph/9809583

The resolution of the horizon problem, which underlies inflationary cosmology [1], is that at a very early time, the equation of state that dictates the expansion rate of the Universe was dominated by a vacuum energy density  $\rho_v$ , so that a small causally connected patch could grow to a size that encompasses the comoving volume which becomes the observed universe today.

In the standard (isentropic) inflationary scenarios, the radiation energy density  $\rho_r$  scales with the inverse fourth power of the scale factor, becoming quickly negligible. In such case, a short time reheating period terminates the inflationary period and initiates the radiation dominated epoch. On the other hand, the only condition required by General Relativity for inflation is that  $\rho_r < \rho_v$ . Inflation in the presence of nonnegligible radiation is characterized by non-isentropic expansion [2,3] and thermal seeds of density perturbations [4]. This can be realized in warm inflation scenarios [5] where there is no reheating.

The basic idea of our implementation of warm inflation is quite simple; a scalar field, which we call the inflaton, is coupled to several other fields. As the inflaton relaxes toward its minimum energy configuration, it will decay into lighter fields, generating an effective viscosity. That this indeed happens has been demonstrated in detail in Refs. [6–8]. If this viscosity is large enough, the inflaton will reach a slow-roll regime, where its dynamics becomes overdamped. This overdamped regime has been analyzed in Ref. [9]. As one expects, overdamping is most successful for the case where the inflaton is coupled to a large number of fields which are thermally excited, *i.e.*, which have small masses compared to the ambient temperature of radiation. This result has important consequences for cosmological applications.

In order to satisfy one of the requirements of a successful inflation (60 or so e-folds), overdamping must be very efficient. Our goal in this Letter is to show that it is possible to build a toy model, motivated from high energy particle physics, that can provide enough overdamping as to produce a viable inflationary expansion, which smoothly exits into a radiation-dominated regime, with  $\rho_r$  slowly and monotonically decreasing throughout the whole process. In contrast to most models in the literature, warm inflation provides both a natural context for the slow-roll regime and for its graceful termination into a radiation-dominated era.

The particle physics model considered below is inspired by string theories exhibiting  $N = 1$  global supersymmetry, with the inflaton coupled to massive modes of the string, as recently discussed in Ref. [10]. We refer the reader to this reference for details.

We thus consider the following Lagrangian of a scalar field  $\phi$  interacting with  $N_M \times N_\chi$  scalar fields  $\chi_{jk}$  and  $N_M \times N_\psi$  fermion fields  $\psi_{jk}$ ,

$$\begin{aligned} \mathcal{L}[\phi, \chi_{jk}, \bar{\psi}_{jk}, \psi_{jk}] = & \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 \\ & + \sum_{j=1}^{N_M} \sum_{k=1}^{N_\chi} \left\{ \frac{1}{2}(\partial_\mu \chi_{jk})^2 - \frac{f_{jk}}{4!}\chi_{jk}^4 - \frac{g_{jk}^2}{2}(\phi - M_j)^2 \chi_{jk}^2 \right\} \\ & + \sum_{j=1}^{N_M} \sum_{k=1}^{N_\psi} \{ i\bar{\psi}_{jk} \not{\partial} \psi_{jk} - h_{jk}(\phi - M_j)\bar{\psi}_{jk}\psi_{jk} \}, \end{aligned} \quad (1)$$

where all coupling constants are positive:  $\lambda, f_{jk}, g_{jk}^2, h_{jk} > 0$ . For simplicity, we consider in the following  $f_{jk} = f, g_{jk} = h_{jk} = g$ . Also, we will set  $N_\psi = N_\chi/4$ , which along with our choice of coupling implies a cancellation of radiatively generated vacuum energy corrections in the effective potential [11]. We call this kind of model a distributed mass model (DMM), where the interaction between  $\phi$  with the  $\chi_{jk}$  and  $\psi_{jk}$  fields establishes a mass scale distribution for the  $\chi_{jk}$  and  $\psi_{jk}$  fields, which is determined by the mass parameters  $\{M\}$ . Thus the  $\chi_{jk}$  and  $\psi_{jk}$  effective field-dependent masses,  $m_{\chi_{jk}}(\phi, T, \{M\})$  and  $m_{\psi_{jk}}(\phi, T, \{M\})$ , respectively, can be constrained even when  $\langle \phi \rangle = \varphi$  is large. A specific choice of  $\{M\}$  will be given shortly. The  $\phi\chi, \phi\psi$  interactions can be made reflection symmetric,  $\phi \rightarrow -\phi$ , but for our purposes we will consider all  $M_j > 0$  and  $\phi > 0$ .

The 1-loop effective equation of motion for the scalar field  $\phi$  is obtained by setting  $\phi = \varphi + \eta$  in Eq. (1) and

imposing  $\langle \eta \rangle = 0$ . Then from Weinberg's tadpole method [12,13,9] the 1-loop evolution equation for  $\varphi$  is

$$\begin{aligned} \ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi + \frac{\lambda}{6}\varphi^3 + \frac{\lambda}{2}\varphi\langle\eta^2\rangle \\ + g^2 \sum_i^{N_M} \sum_j^{N_\chi} (\varphi - M_i) \langle \chi_{ij}^2 \rangle + g \sum_i^{N_M} \sum_j^{N_\chi/4} \langle \psi_{ij} \bar{\psi}_{ij} \rangle = 0. \end{aligned} \quad (2)$$

In the above, the term  $3H\dot{\varphi}$  describes the energy red-shift of  $\varphi$  due to the expansion of the Universe. In the warm-inflation regime of interest here, the characteristic time scales (given by the inverse of the decay width) for the fields in Eq. (1) are faster than the expansion time scale,  $1/H$ . In this case, the calculation of the (renormalized) thermal averages in Eq. (2) can be approximated just as in the Minkowski spacetime case.

A systematic evaluation of the averages in the adiabatic, strong dissipative regime was presented in [9] and re-derived in [14] with extension to fermions. The essential mechanism for dissipation obtained from this approach can be explained through an intuitive kinetic theory argument first given in [7] and reexamined in [14]. (We note that the objections to warm inflation raised in Ref. [14] are avoided quite naturally by coupling the inflaton to a tower of mass modes as in the present implementation of the DM model.) For the  $\chi$ -field averages one writes  $\langle \chi_{ij}^2 \rangle = \int (d^3k/(2\pi)^3) n_{\chi_{ij}}(\mathbf{k})/\omega_{\chi_{ij}}(\mathbf{k})$ , where the number densities  $n(\mathbf{k})$  in the strong dissipative regime, in near equilibrium, can be written in the relaxation time approximation to the kinetic equation as [7,15]

$$n(\mathbf{k}) \sim n^{\text{eq.}}(\mathbf{k}) - \tau \dot{n}^{\text{eq.}}(\mathbf{k}). \quad (3)$$

Here  $n^{\text{eq.}}(\mathbf{k}) = 1/(e^{\beta\omega(\mathbf{k})} - 1)$  is the equilibrium number density for  $\chi$  particles,  $\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m^2(\varphi, T, \{M\})}$ ,  $m^2(\varphi, T, \{M\})$  is the effective, field dependent mass for the  $\chi$ -fields [9], and  $\tau = \Gamma^{-1}$ , where  $\Gamma$  is the decay width for the  $\chi$ -particles. From Eq. (3) the second term on the right is proportional to  $\dot{\varphi}$ , which in Eq. (2) leads to dissipative effects on  $\varphi$  from its interaction with the  $\chi$ -fields. Analogous expressions also apply to the fermionic averages (for Fermi-Dirac statistics) in Eq. (2).

Based on a systematic perturbative approach, as we have shown in previous work [6,9], we can write Eq. (2), using the expressions for the associated averages for the  $\chi_{ij}$ ,  $\psi$  and  $\eta$  fields and Eq. (3), as

$$\ddot{\varphi} + 3H\dot{\varphi} + V'_{\text{eff}}(\varphi, T) + \eta(\varphi)\dot{\varphi} = 0, \quad (4)$$

where  $V'_{\text{eff}}(\varphi, T) = \partial V_{\text{eff}}(\varphi, T)/\partial\varphi$ , is the field derivative of the 1-loop finite temperature effective potential, which can be computed by the standard methods (in fact this is the resummed effective potential, with masses written in terms of the finite temperature effective masses) and  $\eta(\varphi) \equiv \eta^{\text{B}}(\varphi) + \eta^{\text{F}}(\varphi)$  is a field dependent dissipation,

$$\begin{aligned} \eta^{\text{B}}(\varphi) = \frac{\lambda^2}{8} \varphi^2 \frac{1}{\beta^{-1}} \int \frac{d^3q}{(2\pi)^3} \frac{n_\phi^{\text{eq.}}(1 + n_\phi^{\text{eq.}})}{\omega_\phi^2(\mathbf{q}) \Gamma_\phi(\mathbf{q})} \\ + \sum_{i=1}^{N_M} \sum_{j=1}^{N_\chi} \frac{g^4}{2} \frac{(\varphi - M_i)^2}{\beta^{-1}} \int \frac{d^3q}{(2\pi)^3} \frac{n_{\chi_{ij}}^{\text{eq.}}(1 + n_{\chi_{ij}}^{\text{eq.}})}{\omega_{\chi_{ij}}^2(\mathbf{q}) \Gamma_{\chi_{ij}}(\mathbf{q})} \end{aligned} \quad (5)$$

and

$$\eta^{\text{F}}(\varphi) = \sum_{i=1}^{N_M} \sum_{j=1}^{N_\chi/4} g^2 \frac{(\varphi - M_i)^2}{\beta^{-1}} \int \frac{d^3q}{(2\pi)^3} \frac{n_{\psi_{ij}}^{\text{eq.}}(1 - n_{\psi_{ij}}^{\text{eq.}})}{\omega_{\psi_{ij}}^2(\mathbf{q}) \Gamma_{\psi_{ij}}(\mathbf{q})}. \quad (6)$$

In what follows, we will be interested in the regime where  $\eta(\varphi) \gg 3H$  in Eq. (4). As discussed in [7,9,14], Eq. (4) is valid provided the adiabatic regime for  $\varphi$  is satisfied, *i.e.*, the dynamic time-scale for  $\varphi$  must be much larger than the typical collision time-scale ( $\sim \Gamma^{-1}$ ), or  $|\varphi/\dot{\varphi}| \gg \Gamma^{-1}$ , where  $\Gamma$  is the smallest of the thermal decay widths  $\Gamma_\phi$ ,  $\Gamma_{\chi_{ij}}$ ,  $\Gamma_{\psi_{ij}}$ , as it will set the largest time-scale for collisions for the system in interaction with the bath.

Note that the damping coefficient depends on  $\varphi^2$  or  $(\varphi - M_i)^2$ . To obtain enough inflation, the amplitude of the inflaton should decrease very slowly, which requires efficient overdamping. This overdamping is guaranteed by having a succession of fields always thermalized, so that the population of decay products is not depleted by the expansion. This condition is satisfied through our choice of Lagrangian for the DM-model, which naturally guarantees that a population of decay products will be generating efficient damping as  $\varphi$  slowly rolls down.

We construct a warm inflation scenario based on the following DM-model. For definiteness, we will refer all dimensional quantities to  $T_{BI}$ , the temperature of the universe at the beginning of warm inflation. The crucial property of the DM model of Eq. (1) is that for a given temperature  $T$ , only the fields with masses  $g^2(\varphi - M_i)^2 \lesssim T^2$  will contribute to the thermal viscosity; the effect of heavier fields can be neglected. Thus, as the inflaton rolls down its potential, we only must consider the subset of fields for values of  $i$  which satisfy the above inequality. Note that with this choice of model, if  $\varphi \gg T_{BI}$ , which is needed for efficient inflation, it is still possible to have many light  $\chi_{ij}$  ( $\psi_{ij}$ ) fields if  $\varphi \sim M_i$ . This will guarantee an efficient viscosity term in the equation for  $\varphi$ .

For convenience, we write the mass parameters as  $M_i = (i + i_{\text{min}}) \bar{m}_{\chi\psi}^{\text{max}} T_{BI}/g$ ,  $i = 1, \dots, N_M$ , with  $\bar{m}_{\chi\psi}^{\text{max}}$  a dimensionless constant of  $\mathcal{O}(1)$  and  $M_{i=0} = i_{\text{min}} T_{BI}/g$  the lowest mass level coupled to  $\varphi$ . For such a model, at  $T \sim T_{BI}$  and  $M_{i+1} > \varphi > M_i$ , there will be a range of masses when  $2.5N_\chi + 1$   $\chi_{ij}$ ,  $\psi_{ij}$ ,  $\chi_{i+1,j}$ ,  $\psi_{i+1,j}$ , and  $\eta$  fields are thermally excited. All other  $\chi_{i'j}$  ( $\psi_{i'j}$ )-fields, for  $i' \neq i, i+1$  and  $j = 1, \dots, N_\chi$  ( $N_\chi/4$ ), have masses  $m_{\chi\psi}^2 \approx g^2(\varphi - M_{i'})^2 > (\bar{m}_{\chi\psi}^{\text{max}} T_{BI})^2 \sim T^2$  and are thus cold (actually thermal excitation begins once  $m_{\chi\psi} \approx 10T$  and nontrivial dissipation once  $m_{\chi\psi} \approx (2-3)T$ ).

For the  $\eta - \chi - \psi$  system participating in the dissipative dynamics of  $\varphi$ , in each interval  $M_{i+1} > \varphi > M_i$ ,

the radiation energy density is  $(2N_\chi + 7N_\psi + 1)\pi^2 T^4/30$ . In addition to these fields, in general there can be a set of “heat bath” fields that increase the particle degrees in the radiation system, but do not otherwise contribute to dissipation. For later use, we adopt the following notation for the number of these heat bath fields,  $2(15N_\chi/8+1)^{1+\alpha} - (15N_\chi/4+1)$ , so that, in total, the radiation system has  $g_* \equiv 2(15N_\chi/8+1)^{1+\alpha}$  particle degrees for any interval where  $\varphi$  is.  $\alpha \geq 0$  is a free parameter.

To simplify the calculation to follow, we consider the region where  $\varphi \gg T$  and the  $\lambda\varphi^3/6$  term dominates the equation of motion. From  $V'_{\text{eff}}(\varphi, T)$  the leading  $\chi, \psi$  field contribution for  $M_{i+1} > \varphi > M_i$  is  $(N_\chi T_{BI}^2/8)g^2[(\varphi - M_i) + (\varphi - M_{i+1})] < \mathcal{O}(gN_\chi T^3)$ , so that the constraint requires  $\lambda\varphi^3 \gg \mathcal{O}(gN_\chi T^3)$ . In the perturbative regime that we examine,  $\lambda \ll 1$ , so that  $m_\phi^2(\varphi, T) \approx \lambda\varphi^2/2 \equiv \bar{m}_\phi^2(\phi, T)T_{BI}^2$ , since  $\varphi \gg T$ . The  $\chi(\psi)$ -masses will range from  $f^2 T^2/12$  ( $g^2 T^2/6$ )  $< m_\chi^2(m_\psi^2) < f^2 T^2/12$  ( $g^2 T^2/6$ )  $+ g^2(\varphi - M_i)^2 \lesssim \mathcal{O}(T_{BI}^2)$ . As a simplification, the  $\chi, \psi$ -masses will be estimated at their largest value  $(m_{\chi\psi}^{\text{max}})^2 \equiv (\bar{m}_{\chi\psi}^{\text{max}} T_{BI})^2 \approx g^2(\varphi - M_i)^2|_{\text{max}} \approx \mathcal{O}(T_{BI}^2)$ . We can then express the condition that the  $\lambda\varphi^3/6$  term dominates the equation of motion Eq. (4), in terms of

$$R_{\chi\psi/\varphi} \leq \frac{3gN_\chi m_{\chi\psi}^{\text{max}} T_{BI}^2}{4\lambda\varphi_{BI}^3} < 1, \quad (7)$$

where,  $R_{\chi\psi/\varphi} \equiv \frac{3g^2 N_\chi T_{BI}^2 [(\varphi - M_i) + (\varphi - M_{i+1})]_{\text{max}}}{4\lambda\varphi_{BI}^3}$ .

To impose the most stringent constraint from this, it will be taken at the maximum value  $m_{\chi\psi}^{\text{max}}$  from Eq. (7). In fact, considerable increase in dissipation, thus improvement in the results to follow, occur by accounting for corrections when the  $\chi$ 's ( $\psi$ 's) are in the smaller mass region. In the regime outlined above, the effective equation of motion for  $\varphi$ , Eq. (4), in the interval  $M_{i+1} > \varphi > M_i$  and in the overdamped limit is

$$\eta_{i,i+1}(\varphi)\dot{\varphi} \simeq -\frac{\lambda}{6}\varphi^3, \quad (8)$$

where  $\eta_{i,i+1}(\varphi) \equiv \eta_1^B [(\varphi - M_i)^2 + (\varphi - M_{i+1})^2] + \eta_1^F T^2$  where  $\eta_1^B$  and  $\eta_1^F$  (from [9] and [14]) are given by:  $\eta_1^B \sim 384N_\chi g^4/[\pi T(f^2 + 8g^4)] \ln(2T/m_\chi)$  and  $\eta_1^F \sim 11N_\psi/T$ , respectively.

As  $\varphi$  moves through the interval  $M_{i+1} > \varphi > M_i$ ,

$$\eta_{i,i+1}(\varphi) = \kappa(1 + r_{\text{FB}})\eta_1^B(\bar{m}_{\chi\psi}^{\text{max}} T_{BI})^2/g^2, \quad (9)$$

with  $r_{\text{FB}} \equiv \eta_1^F T_{BI}^2 g^2/[\eta_1^B \kappa(\bar{m}_{\chi\psi}^{\text{max}} T_{BI})^2] \approx 0.2$ , where the estimate on the right is from [9,14] for the high temperature limit and  $0.5 < \kappa < 1$ . Since we are examining the region  $\varphi \gg T$ , we can ignore the weak  $\varphi$  dependence in  $\eta_{i,i+1}$ . In this case, the solution to Eq. (8) is

$$\varphi(\tau) = \varphi_{BI} \left[ \frac{\tau + \tau_0}{W_4} + 1 \right]^{-1/2}, \quad (10)$$

with  $W_4 = 3H_{BI}\kappa\eta_1^B(\bar{m}_{\chi\psi}^{\text{max}} T_{BI})^2(1 + r_{\text{FB}})/(g^2\lambda\varphi_{BI}^2)$  and  $H_{BI} = \sqrt{2\pi\lambda\phi_{BI}^4/(9m_p^2)}$ . Time has been rescaled as  $t = \tau/H_{BI}$  with the origin chosen such that  $\varphi(\tau = 0) \equiv \varphi_{BI}$ , which implies  $\tau_0 = 0$ .

The resulting warm inflation cosmology for such a field trajectory in any interval  $M_{i+1} > \varphi > M_i$  is equivalent to the  $n = 4$  model in [3]. It yields a power-law warm inflation which never terminates and for which in the steady state regime  $\rho_r(t)/\rho_v(t) = \text{const}$ . In our model, warm inflation terminates into a radiation dominated regime once  $\varphi < M_{i=0}$ , since below that point the dissipative term becomes negligible, in which case  $\varphi$  coasts down the potential. The essential point of interest here is to show that once  $\varphi$  reaches  $M_0$ , sufficient e-folds  $N_e$  have occurred while the universe has nonnegligible radiation.

For simplicity, the steady state cosmology in flat spatial geometry will be examined, which implies from [3], for  $W_4 \gg 1$ ,  $\rho_r(0)/\rho_v(0) = \rho_r(\tau)/\rho_v(\tau) = 1/(2W_4)$ . [From Eq. (10), we can show that this is the necessary condition to satisfy the adiabatic condition,  $|\dot{\varphi}/\varphi| \gg \Gamma^{-1}$  and  $\Gamma\phi_{(\chi)} > H$ .] In terms of the parameters of the model this can also be written as  $g_*\pi^2 T_{BI}^4/30 = \lambda\varphi_{BI}^4/(48W_4)$ . Initial conditions that are more realistic, such as  $\rho_r(0) > \rho_v(0)$  rapidly evolve into the steady state behavior. In this steady state regime, the scale factor solution is [3]

$$R(\tau) = (\tau/W_4 + 1)^{W_4 + \frac{1}{4}}, \quad (11)$$

with initialization  $R(0) = 1$ .

Our goal is to compute  $W_4$  from the microscopic parameters of the model and consistent with the many constraints given above and in section V of [9]. The power-law expansion behavior of the scale factor, Eq. (11), is such that the major factor of growth happens for  $\tau/W_4 < 10$ .  $N_e = W_4$  e-folds occur at time  $\tau/W_4 = e - 1$ . We will restrict our calculation within this time interval. At the end of this interval  $\varphi(\tau)$ , Eq. (10), and  $T$  have not changed significantly,  $\varphi_{BI} \rightarrow \varphi_{BI}/\sqrt{e}$ ,  $T_{BI} \rightarrow T_{BI}/\sqrt{e}$ . This simplifies the constraint equations, since they can be analyzed at the initial values  $\varphi_{BI}$ ,  $T_{BI}$  and approximately will be valid throughout this interval.

The constraint equations for computing  $W_4$  are as follows. To satisfy the thermalization conditions  $\Gamma_\chi, \Gamma_\phi, \Gamma_\psi > H$ , we will set  $H_{BI} = \min(\Gamma_\chi, \Gamma_\phi, \Gamma_\psi)$ . More explicitly, for the warm inflation solutions studied below,  $\Gamma_\chi$  (which is the smallest of the  $\Gamma$ s) is larger than  $H_{BI}$  for  $f \gtrsim 0.8$ . This condition may be under restrictive to obtain good thermalization, but it provides the maximum parameter region that may be useful. It should be noted that since  $H \sim \varphi^2$  and  $\Gamma \sim T$ , as warm inflation proceeds, thermalization improves. The thermalization condition automatically implies that the adiabatic condition is comfortably satisfied.

All the constraints are expressed in the following four relations

$$R_{\chi\psi/\varphi}^2 = \frac{15g^4 N_\chi^2 \left(\frac{15N_\chi}{8} + 1\right)^{-(1+\alpha)}}{256\pi^2 \kappa [\min(\Gamma_\phi, \Gamma_\chi)] \eta_1^B (1 + r_{\text{FB}})}, \quad (12)$$

$$W_4 = \frac{45 \left(g N_\chi \bar{m}_{\chi\psi}^{\text{max}}\right)^2}{512\pi^2 \bar{m}_{\phi_{BI}}^2 R_{\chi\psi/\varphi}^2 \left(\frac{15N_\chi}{8} + 1\right)^{1+\alpha}}, \quad (13)$$

$$R_{\chi\psi/\varphi} \varphi_{BI}/T_{BI} = 3g N_\chi \bar{m}_{\chi\psi}^{\text{max}} / (8\bar{m}_{\phi_{BI}}^2) \quad (14)$$

and

$$\lambda R_{\chi\psi/\varphi}^{-2} = 128\bar{m}_{\phi_{BI}}^6 (3g N_\chi \bar{m}_{\chi\psi}^{\text{max}})^{-2}. \quad (15)$$

Eq. (12) is obtained from the relation for  $W_4$  below Eq. (10), where  $\eta(\varphi)$  in Eq. (9), has been expressed in terms of  $R_{\chi\phi}$ , Eq. (7). The procedure is to input  $g, f, \bar{m}_{\chi\psi}^{\text{max}}, \bar{m}_{\phi_{BI}}, N_\chi$  on the right-hand-side of the above four equations, then obtain the left-hand-side of Eqs. (12) and (14) from which the remaining two equations follow, up to a choice for  $R_{\chi\psi/\varphi} < 1$ . Note that the only cases requiring additional heat bath fields ( $\alpha > 0$ ) are in parametric regimes when the right-hand-side of Eq. (12) is greater than 1, since we require  $R_{\chi\psi/\varphi} < 1$ .

In [9] analytic expressions were obtained for  $\Gamma(\mathbf{q})$  and  $\eta_1$  in the simplified limit  $|\mathbf{q}| = 0$ ,  $m_\chi, m_\psi \sim \mathcal{O}(m_\phi)$  (which here we call level 1) as well as in terms of the exact 2-loop expressions, which must be computed numerically (level 2). The results in Fig. 1 present both levels of approximation. We used  $g = f = 0.9$ ,  $\kappa = 0.5$ ,  $N_\chi = 12$ ,  $N_\psi = N_\chi/4 = 3$  and  $R_{\chi\psi/\varphi} \lesssim 1$  ( $\alpha \gtrsim 0$ ). The solid and dashed curves are for  $\bar{m}_{\chi\psi}^{\text{max}} = 0.9$ , for level 1 and level 2, respectively, and the dotted curve is level 2 for  $\bar{m}_{\chi\psi}^{\text{max}} = 2.5$ . For both curves drawn in Fig. 1, going from  $\bar{m}_{\phi_{BI}} = 0.002$  to 0.05, the initial field displacement  $\varphi_{BI}/T_{BI}$  ranges as  $10^6 - 10^3$ ,  $N_M$  ranges as  $10^5 - 10^2$  and  $\lambda$  ranges as  $\sim 10^{-17} - 10^{-9}$ . In the regime we are considering,  $i_{\text{min}} \sim \varphi_{BI}/(gT_{BI})$  and for the above results,  $i_{\text{min}}$  ranges from  $10^2 - 10^6$ . To obtain  $N_e \sim 60$  e-folds of warm inflation for all three cases,  $\varphi_{BI}/T_{BI} \approx 3000$ ,  $N_M = 1000$ ,  $\lambda \approx 10^{-9}$ . An absolute scale is fixed by setting  $m_p = 10^{19}\text{GeV}$  from which for  $N_e \sim 60$  e-folds, we find  $T_{BI} \sim (10^{13} - 10^{14})\text{GeV}$  and  $H_{BI} \sim (10^9 - 10^{10})\text{GeV}$ . [The temperature at the onset of warm inflation,  $\rho_v = \rho_r$ , is  $(2W_4)^{1/4} \sim 3.3$  times bigger than  $T_{BI}$ , and rapidly decreases to  $T_{BI}$  during the transient period.]

The DM-model studied here was motivated by the requirements of warm inflation, dissipative dynamics and perturbative renormalizability. We can justify our choice of Lagrangian by noting that string-inspired models can display an inflaton coupled to mass modes of a string, as explained in Ref. [10]. Within this context, the large number of fields necessary to realize sufficient inflation is a natural consequence of the modifications of short-distance physics required by string theories.

In summary, first principles quantum field interactions can realize a warm inflation regime with sufficient duration to solve the horizon/flatness problems. The interplay between inflationary expansion and radiation production has been a persistent problem since the earliest history of inflationary cosmology. Thus, despite the many questions opened by our model, its underlying mechanism is a unique and simple resolution to the problem. Further study of the inflaton  $k$ -modes is necessary to address the density fluctuation problem.

We thank A. Linde and J. Yokoyama for their interest. AB was supported by the U.S. Department of Energy. MG was partially supported by the NSF through a Presidential Faculty Fellows Award no. PHY-9453431 and by a NASA grant no. NAG5-6613. ROR was partially supported by CNPq and FAPERJ.

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- [1] A. H. Guth, Phys. Rev. **D23**, 347 (1981); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982); A. Linde, Phys. Lett. **108B**, 389 (1982). For more references see, K. A. Olive, Phys. Rep. **190**, 307 (1990).
  - [2] H. P. de Oliveira and R. O. Ramos, Phys. Rev. **D57**, 741 (1998); E. Gunzig, R. Maartens, and A. V. Nesteruk, Class. Quant. Grav. **15**, 923 (1998).
  - [3] A. Berera, Phys. Rev. **D55**, 3346 (1997).
  - [4] A. Berera and L. Z. Fang, Phys. Rev. Lett. **74**, 1912 (1995).
  - [5] A. Berera, Phys. Rev. Lett. **75**, 3218 (1995); A. Berera, Phys. Rev. **D54**, 2519 (1996).
  - [6] M. Gleiser and R. O. Ramos, Phys. Rev. **D50**, 2441 (1994).
  - [7] A. Hosoya and M. Sakagami, Phys. Rev. **D29**, 2228 (1984).
  - [8] M. Morikawa, Phys. Rev. **D33**, 3607 (1986).
  - [9] A. Berera, M. Gleiser and R. O. Ramos, Phys. Rev. **D58**, 123508 (1998).
  - [10] A. Berera and T. W. Kephart, In Press Phys. Lett. B, 1999, **hep-ph/9811295**.
  - [11] L. Dolan and R. Jackiw, Phys. Rev. **D9**, 3320 (1974).
  - [12] G. Semenoff and N. Weiss, Phys. Rev. **D31**, 699 (1985).
  - [13] D. Boyanovsky *et al.*, Phys. Rev. **D51**, 4419 (1995).
  - [14] J. Yokoyama and A. Linde, **hep-ph/9809409**.
  - [15] M. Morikawa and M. Sasaki, Phys. Lett. **165B**, 59 (1985).

#### FIGURE CAPTIONS

**Figure 1:** Number of e-folds of warm inflation  $N_e$  versus  $m_{\phi_{BI}}$  for  $\bar{m}_{\chi\psi}^{\text{max}} = 0.9$  level 1 (solid), 0.9 level 2 (dashed), and 2.5 level 2 (dotted) with  $g = f = 0.9$ ,  $\kappa = 0.5$ ,  $N_\chi = 12$ , and  $N_\psi = N_\chi/4 = 3$ .

