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## Peculiar Velocities of Nonlinear Structure: Voids in McVittie Spacetime

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### ABSTRACT

As a study of peculiar velocities of nonlinear structure, we analyze the model of a relativistic thin-shell void in the expanding universe. (1) Adopting McVittie (MV) spacetime as a background universe, we investigate the dynamics of an uncompensated void with negative MV mass. Although the motion itself is quite different from that of a compensated void, as shown by Haines & Harris (1993), the present peculiar velocities are not affected by MV mass. (2) We discuss how precisely the formula in the linear perturbation theory applies to nonlinear relativistic voids, using the results in (1) as well as the previous results for the homogeneous background (Sakai, Maeda, & Sato 1993). (3) We re-examine the effect of the cosmic microwave background radiation. Contrary to the results of Pim & Lake (1986, 1988), we find that the effect is negligible. We show that their results are due to inappropriate initial conditions. Our results (1)-(3) suggest that the formula in the linear perturbation theory is approximately valid even for nonlinear voids.

*Subject headings:* large scale structure of universe, cosmology, relativity

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## 1. Introduction

Measurements of large-scale peculiar velocities can provide a constraint on the universe model (see, e.g. Zehavi & Dekel (1999)). In contrast to other cosmological tests, they give a constraint on the density parameter  $\Omega_0$  alone, almost independent of the cosmological constant  $\lambda_0$  (Carroll, Press, & Turner 1992).

Although the relation between the peculiar velocity and the density parameter  $\Omega_0$  is usually given in the linear perturbation theory (LPT) (Peebles 1976), the observed universe has nonlinear density profiles. In fact, a network of nonlinear voids filling the entire universe has been suggested by redshift surveys such as the CfA2 (Geller & Huchra 1989) and the SSRS2 (da Costa et al. 1994). Moreover, using such redshift surveys, the description of a void-filling universe was confirmed quantitatively (El-Ad & Piran 1997; El-Ad, Piran, & da Costa 1996, 1997). The relation between  $\Omega_0$  and peculiar velocities inside underdense regions suggests  $\Omega_0 \leq 0.3$  can be ruled out at the 2.4-sigma level (Dekel & Rees 1994).

It is therefore important to investigate peculiar velocities of nonlinear void structure. Here we consider the model of a relativistic thin-shell void. The expansion law of relativistic voids was investigated originally by Maeda & Sato (1983a,b), developing the metric junction method proposed by Israel (1966). They found analytically that, in the flat universe, the shell radius  $R$  expands asymptotically as  $R \propto t^{(15+\sqrt{17})/24} \approx t^{0.797}$  (Maeda & Sato 1983a). For the self-similar void model, on the other hand, Bertschinger (1985) obtained the solution with  $R \propto t^{0.8}$ . The difference of the two results is so small that we do not have to care which model is better, which is physically determined by the radiative process in the shell. For other universe models, the motion of the shell was calculated numerically (Maeda & Sato 1983b): in the open universe, the shell expansion is eventually frozen to the background expansion; on the other hand, in the closed universe, the shell expands much faster and its velocity finally approaches the speed of light. The relation between the peculiar velocity of the shell and the universe model was later investigated systematically (Sakai, Maeda, & Sato 1993).

Lake & Pim extended the work of Maeda & Sato so as to include a mass inside a void (Lake & Pim 1985) and the cosmic microwave background (CMB) radiation (Pim & Lake 1986, 1988). In particular, they claimed that the inclusion of radiation has significant quantitative and qualitative effects on the evolution of the void. It was shown, for instance, that the asymptotic behavior of the shell is  $R \propto t$  in the flat universe if the CMB radiation is included (Pim & Lake 1986). This result is in contrast to that for the vacuum void ( $R \propto t^{0.8}$ ), and hence quite surprising.

Haines & Harris (1993), on the other hand, included a mass *outside* a void by employing McVittie (1966, hereafter MV) metric instead of Friedmann-Robertson-Walker (FRW) metric. MV metric approximates a spherical mass embedded in an asymptotically FRW spacetime. “MV mass” represents the degree to which the void is not compensated by the mass of the shell. They demonstrated the history of the shell in the flat MV spacetime, showing that the negative MV mass acts to accelerate the shell expansion.

In this paper we extend the previous work to clarify the following points.

(1) Peculiar velocities of uncompensated void, which is characterized by negative MV mass. Although Haines & Harris (1993) discussed the dynamics of voids with non-zero MV mass, the effect of MV mass on the peculiar velocity is not clear. It is important to see it because there is no evidence that actual voids have the shells with compensated mass. For example, if voids originate from primordial bubbles that are nucleated in a phase transition during inflation (see, e.g. Amendola et al. (1996)), it is unlikely that voids have compensated shells.

(2) The relation between  $\Omega_0$  and peculiar velocities was derived in LPT (Peebles 1976). It is important to see how precisely the formula applies to nonlinear relativistic voids. We address this question, using the results obtained in (1) as well as the previous results (Sakai, Maeda, & Sato 1993).

(3) As we mentioned above, Pim & Lake (1986, 1988) arrived at the surprising conclusion that the effect of CMB radiation is significant. If it is true, we should take it into account seriously when we constrain the cosmological parameters from the observation of bulk flow. Therefore, their result deserves closer examination.

This paper is organized as follows. In section 2, we present the relativistic equations of motion for a thin shell in MV spacetime, which will be solved later numerically. In section 3, we investigate how the peculiar velocity changes due to MV mass for uncompensated voids. In section 4, we compare the results for the present model and those in the LPT. In section 5, we examine the effect of the CMB radiation on the void evolution. These results are summarized in section 6.

## 2. Basic Equations

### 2.1. McVittie Spacetime

The spacetime described by MV metric has several useful properties:

- The near-field limit is Schwarzschild, in isotropic coordinates;
- The far-field is a FRW spacetime;
- The energy-momentum tensor has a perfect-fluid form.

It is an exact embedding of the Schwarzschild metric into the FRW metric.

The line element is

$$ds^2 = - \left( \frac{1-h}{1+h} \right)^2 dt^2 + a^2(t)(1+h)^4 \{ d\chi^2 + f^2(\chi)(d\theta^2 + \sin^2 \theta d\varphi^2) \}, \quad (2-1)$$

where

$$f(\chi) \equiv \begin{cases} \sin \chi & (k = +1) \\ \chi & (k = 0) \\ \sinh \chi & (k = -1) \end{cases} \quad \text{and} \quad h \equiv \frac{m}{4a(t)f(\chi/2)} \quad (2-2)$$

The Einstein equations yield

$$\frac{8\pi G\rho}{3} = H^2 + \frac{k}{a^2(1+h)^5}, \quad (2-3)$$

$$8\pi Gp = \frac{1}{1-h} \left[ -\frac{2(1+h)}{a} \frac{d^2a}{dt^2} - (1-5h)H^2 - \frac{k}{a^2(1+h)^5} \right], \quad (2-4)$$

where  $\rho$  and  $p$  are the energy density and the pressure, respectively, which are inhomogeneous in general.  $H \equiv (da/dt)/a$  is the Hubble parameter at  $\chi \rightarrow \infty$ .  $m$  is a constant and called MV mass. The  $h \rightarrow 0$  limit is clearly FRW, while Schwarzschild solution is recovered by  $a \rightarrow 1$ . If  $k = 0$  or  $-1$ , the scale factor  $a(t)$  always takes the Friedmann solution.

To see the meaning of MV mass, let us calculate the local gravitational mass defined by Misner & Sharp (1964):

$$M \equiv \frac{R}{2G} (1 - g^{\mu\nu} \partial_\mu R \partial_\nu R) \quad \text{with} \quad R \equiv \sqrt{g_{\theta\theta}}. \quad (2-5)$$

For each  $k$  we find

$$M(R) = \frac{4\pi}{3} \rho R^3 + my(\chi) \quad \text{with} \quad y(\chi) = \begin{cases} \cos^5(\chi/2) & (k = +1) \\ 1 & (k = 0) \\ \cosh^5(\chi/2) & (k = -1) \end{cases} \quad (2-6)$$

As long as the void's size is much smaller than the horizon scale,  $\chi \ll 1$  and  $y(\chi) \approx 1$  for any background model. In this limit, we may therefore interpret the MV mass as approximately the Misner-Sharp mass minus the background mass  $(4\pi/3)\rho R^3$ .

For a thorough discussion of the McVittie metric, its history, and its place among inhomogeneous models, see Krasinski (1997).

## 2.2. Junction Conditions

Let us derive the equations of motion for a spherical shell around a void, by developing the thin-shell formalism of Israel (1966). The basic equations for the shell in the flat MV spacetime were given by Haines & Harris (1993). Here we rewrite the equations as in a simpler form, which also describe the shell in a closed or open background, by extending the equations of Sakai, Maeda, & Sato (1993). Because we are interested only in the effect of the outer MV mass, we assume the inside region to be homogeneous throughout the paper.

Let a time-like hypersurface  $\Sigma$ , which denotes the world-hypersurface of a spherical shell, divide a spacetime into two regions,  $V^+$  (outside) and  $V^-$  (inside). We define a unit space-like vector  $N_\mu$ ,

which is orthogonal to  $\Sigma$  and pointing from  $V^-$  to  $V^+$ . It is convenient to introduce a Gaussian normal coordinate system  $(n, x^i)$ <sup>2</sup> in such a way that the hypersurface of  $n = 0$  corresponds to  $\Sigma$ . From the assumption that a shell is infinitely thin, the surface energy-momentum tensor of the shell is defined as

$$S_{\mu\nu} \equiv \lim_{\epsilon \rightarrow 0} \int_{-\epsilon}^{\epsilon} T_{\mu\nu} dn. \quad (2-7)$$

Using the extrinsic curvature tensor of the hypersurface of the shell,  $K_{ij} \equiv N_{i;j}$ , and the Einstein equations, we can write down the jump conditions on the shell as (Berezin, Kuzmin, & Tkachev 1987)

$$[K_{ij}]^{\pm} = -8\pi G \left( S_{ij} - \frac{1}{2} h_{ij} \text{Tr} S \right), \quad (2-8)$$

$$-S^j_i|_j = [T^j_i]^{\pm}, \quad (2-9)$$

$$\frac{K_j^{i+} + K_j^{i-}}{2} S^j_i = [T^n_i]^{\pm}, \quad (2-10)$$

where  $h_{ij}$  denotes the three metric of  $\Sigma$ , and  $|$  denotes the covariant derivative with respect to  $h_{ij}$ . We have denoted the value of any field variable  $\Psi$  on  $\Sigma$  on the side of  $V^{\pm}$  by  $\Psi^{\pm}$  and defined a bracket as  $[\Psi]^{\pm} \equiv \Psi^+ - \Psi^-$ . Eliminating  $K_{ij}^-$  from equations (2-8) and (2-10), we can derive the equation (Berezin, Kuzmin, & Tkachev 1987):

$$K_j^{i+} S^j_i + 4\pi G \left\{ S^j_i S^j_i - \frac{1}{2} (\text{Tr} S)^2 \right\} = [T^n_i]^{\pm}. \quad (2-11)$$

If the outer region is homogeneous, equation (2-11) leads to a simple expression of the basic equation, because it does not contain the metric in  $V^-$  Sakai & Maeda (1993a,b); Sakai, Maeda, & Sato (1993). On the other hand, if we assume the inside region to be homogeneous, which is the case here, it is easier to solve the equation obtained by eliminating  $K_{ij}^+$ :

$$K_j^{i-} S^j_i - 4\pi G \left\{ S^j_i S^j_i - \frac{1}{2} (\text{Tr} S)^2 \right\} = [T^n_i]^{\pm}. \quad (2-12)$$

The line elements in  $V^+$  and in  $V^-$  are described by

$$ds^2 = - \left( \frac{1-h}{1+h} \right)^2 dt_+^2 + a_+^2(t_+)(1+h)^4 \{ d\chi_+^2 + f_+^2(\chi_+)(d\theta^2 + \sin^2 \theta d\varphi^2) \}, \quad (2-13)$$

$$ds^2 = -dt_-^2 + a_-^2(t_-) \{ d\chi_-^2 + f_-^2(\chi_-)(d\theta^2 + \sin^2 \theta d\varphi^2) \}, \quad (2-14)$$

The direct calculation of  $K_j^{i-}$  yields (Sakai & Maeda 1993a)

$$K_{\theta}^{\theta-} = \frac{\gamma_-(f'_- + v_- H_- R)}{R}, \quad K_{\tau}^{\tau-} = \gamma_-^3 \frac{dv_-}{dt_-} + \gamma_- v_- H_-, \quad (2-15)$$

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<sup>2</sup>In this paper, Greek letters run from 0 to 3, while Latin letters run in 0, 2, 3.

where the circumference radius of the shell  $R$ , the peculiar velocity of the shell  $v_-$ , and its Lorentz factor  $\gamma_-$  are defined as

$$R \equiv a_+ f_+ = a_- f_-, \quad v_- \equiv a_- \frac{d\chi_-}{dt_-}, \quad \text{and} \quad \gamma_- \equiv \frac{1}{\sqrt{1 - v_-^2}}. \quad (2-16)$$

Similarly,  $v_+$  and  $\gamma_+$  in the MV spacetime are defined as

$$v_+ \equiv \frac{a_+(1+h)^3}{1-h} \frac{d\chi_+}{dt_+} \quad \text{and} \quad \gamma_+ \equiv \frac{1}{\sqrt{1 - v_+^2}}. \quad (2-17)$$

As energy-momentum tensors, we consider perfect fluid on  $\Sigma$  and in  $V^\pm$ , i.e.,

$$S_{\mu\nu} = (\sigma + \varpi) v_\mu^\pm v_\nu^\pm + \varpi h_{\mu\nu}, \quad (2-18)$$

$$T_{\mu\nu}^\pm = (\rho^\pm + p^\pm) u_\mu^\pm u_\nu^\pm + p g_{\mu\nu}, \quad (2-19)$$

where  $\sigma$ ,  $\varpi$ ,  $v_\mu$ , and  $u_\mu$  are the surface density, the surface pressure, the four velocity of the shell, and the four velocity of the background fluid, respectively. In the Gaussian normal coordinate system, we have

$$T_n^{n\pm} = \gamma^2(v^2\rho + p)|^\pm, \quad T_\tau^{n\pm} = \gamma^2 v(\rho + p)|^\pm. \quad (2-20)$$

Now, with the help of equations (2-15), (2-18), and (2-20), we can write down equations (2-9) and (2-12) explicitly as

$$\gamma_- \frac{d\sigma}{dt_-} = -2\gamma_- \frac{dR}{dt_-} \frac{\sigma + \varpi}{R} + [\gamma v(\rho + p)]^\pm, \quad (2-21)$$

$$\gamma_-^3 \frac{dv_-}{dt_-} = -\gamma_- \left\{ \left(1 - 2\frac{\varpi}{\sigma}\right) v_- H_- - \frac{2f'_- \varpi}{R \sigma} \right\} - 2\pi G(\sigma + 4\varpi) - \frac{[\gamma^2(v^2\rho + p)]^\pm}{\sigma}. \quad (2-22)$$

The relation between  $dR/dt_-$  and  $v_-$  is given by

$$\frac{dR}{dt_-} = f'_- v_- + H_- R. \quad (2-23)$$

Further, the conditions of the continuity of the metric,

$$d\tau^2 = \left(\frac{1-h}{1+h}\right)^2 dt_+^2 - a_+^2(t_+)(1+h)^4 d\chi_+^2 = dt_-^2 - a_-^2(t_-) d\chi_-^2, \quad (2-24)$$

$$\frac{dR}{d\tau} = \frac{d}{d\tau} \{(1+h)^2 a_+ f_+\} = \frac{d}{d\tau} (a_- f_-), \quad (2-25)$$

reduce to

$$\frac{dt_+}{dt_-} = \frac{1+h}{1-h} \frac{\gamma_+}{\gamma_-}, \quad (2-26)$$

$$\gamma_+ \left\{ \left( f'_+ + \frac{2h'f_+}{1+h} \right) v_+ + H_+ R \right\} = \gamma_- (f'_- v_- + H_- R). \quad (2-27)$$

The equations of motion for the shell are determined by equations (2-21), (2-22), and (2-23). We use the above supplementary equations (2-26) and (2-27) to give  $t_+$  and  $v_+$ , respectively; they are used at the initial time as well as at each step of time evolution.

The angular component of the jump condition (2-8),

$$\gamma_+ \left( f'_+ + \frac{2h'f_+}{1+h} + v_+ H_+ R \right) - \gamma_- (f'_- + v_- H_- R) = -4\pi G \sigma R, \quad (2-28)$$

gives a constraint for the relation between the surface density  $\sigma$  and MV mass  $m$ . We use it for giving initial data as well as for checking numerical errors of integration. For integration we adopt the 4th order Runge-Kutta method. Throughout the analysis we did not encounter any numerical problem: the relative errors of equation (2-28) were always less than  $10^{-13}$ .

The equations of motion presented here and by Sakai, Maeda, & Sato (1993) have several advantages compared with the equations derived in other papers. First, the expression is much simpler. Secondly, there is no sign ambiguity in the relation between  $t_+$  and  $t_-$ , (2-26), contrary to the comment by Pim & Lake (1986). Thirdly, our expression for the extrinsic curvature  $K_\theta^\theta$ , (2-15), can take both positive and negative values without ambiguity. Although our equations make numerical integration easier, they may not be so convenient for analytic arguments.

### 3. Peculiar Velocities of Uncompensated Voids

Here we consider only dust as matter fluid:

$$p_\infty^\pm = 0 \quad \text{and} \quad \varpi = 0, \quad (3-1)$$

where the subscript  $\infty$  denotes quantities at  $\chi_+ \rightarrow \infty$ . A compensated void simply means  $m = 0$ , i.e., the background is described by the FRW metric. On the other hand, an uncompensated void is characterized by negative MV mass. Here we fix the value of  $m$  by supposing no shell ( $\sigma_i = 0$ ) at the initial time  $t_i$ . The initial time  $t_i$  is a free parameter, which is determined by the structure formation model; in the following we set  $t_i$  as the decoupling time, i.e.,  $z_i = 1000$ . The remaining initial parameters are fixed as follows:

$$\begin{aligned} v_i^+ &= 0, \quad R_i H_i^+ = 0.1, \quad H_i^+ = H_i^-, \quad \rho_i^- = 0, \\ \Omega_i &\equiv \frac{8\pi G \rho_i^+}{3H_i^2} = 1 \quad \text{or} \quad 0.98. \end{aligned} \quad (3-2)$$

Figure 1(a) shows the motion of the shell in terms of the comoving coordinate  $\chi$ . As shown by Haines & Harris (1993), negative MV mass pushes the shell faster. Although Figure 1(a) indicates



that the effect of MV mass looks quite large, the behavior of  $\chi$  is not observable. What we can observe is the radius and velocity of the shell at the present epoch. We thus plot the peculiar velocity normalized by the Hubble expansion:

$$\tilde{v} \equiv \frac{v}{HR}. \quad (3-3)$$

in Figure 1(b). The asymptotic behavior is determined by  $\Omega_0$ , independent of MV mass. Figure 2 reports the relation between  $\tilde{v}_0$  and  $\Omega_0$ , which confirms that  $\tilde{v}_0$  does not depend on whether the void is compensated or not.

#### 4. Comparison with Linear Perturbation Theory

In this section we discuss the relation between  $\Omega_0$  and  $\tilde{v}_0$ , comparing the results in the relativistic void model and those in LPT.

The peculiar velocity  $\mathbf{v}$  for general density fluctuations in LPT is (Peebles 1976)

$$\mathbf{v} = \frac{2F\mathbf{g}}{3H\Omega} \quad \text{with} \quad F \approx \Omega^{0.6}, \quad (4-1)$$

where  $\mathbf{g}$  is the peculiar gravitational acceleration. For a spherically symmetric system, the gravitational acceleration is given by

$$g(R) = -\frac{G\delta M(R)}{R^2}, \quad (4-2)$$

where  $\delta M(R)$  is the difference between the mass within a sphere and the unperturbed mass within the sphere with the same radius  $R$ .

For the void model,  $\delta M(R)$  depends on whether we measure it just inside the shell ( $R = R_-$ ) or just outside it ( $R = R_+$ ):

$$\delta M(R_-) = -\frac{4\pi}{3}(\rho^+ - \rho^-)R^3, \quad \delta M(R_+) = -\frac{4\pi}{3}(\rho^+ - \rho^-)R^3 + 4\pi\sigma R^2. \quad (4-3)$$

It is therefore reasonable to define the mass difference as the average:

$$\delta M(R) \equiv \frac{\delta M(R_+) + \delta M(R_-)}{2} = -\frac{4\pi}{3}(\rho^+ - \rho^-)R^3 + 2\pi\sigma R^2. \quad (4-4)$$

On the other hand, one of the junction conditions (2-28) can be rewritten as

$$\begin{aligned} \varepsilon^+ \sqrt{1 + \left(\frac{dR}{d\tau}\right)^2 - \frac{8\pi G\rho^+}{3}R^2 - \frac{2Gmy}{R}} - \varepsilon^- \sqrt{1 + \left(\frac{dR}{d\tau}\right)^2 - \frac{8\pi G\rho^-}{3}R^2} \\ = -4\pi G\sigma R, \end{aligned} \quad (4-5)$$

where  $\varepsilon^\pm \equiv \text{sign} K_\theta^{\theta^\pm}$ . In the Newtonian approximation,  $(dR/d\tau)^2 \ll 1$ ,  $y(\chi) \approx 1$ , and  $\varepsilon^\pm = +1$ , equation (4-5) reduces to mass conservation:

$$m + \frac{4\pi}{3}(\rho^+ - \rho^-)R^3 = 4\pi\sigma R^2. \quad (4-6)$$

From equations (3-3), (4-1), (4-2), (4-4), and (4-6), we obtain

$$\tilde{v} = \frac{\Omega^{0.6}}{6} \left( 1 - \frac{\rho^-}{\rho^+} - \frac{m}{4\pi\rho^+R^3/3} \right). \quad (4-7)$$

For compensated voids ( $m = 0$ ), equation (4-7) reduces to a simple expression:

$$\tilde{v} = \frac{\Omega^{0.6}}{6} \left( 1 - \frac{\rho^-}{\rho^+} \right). \quad (4-8)$$

For uncompensated voids ( $\sigma_i = 0$ ), on the other hand, equations (4-7), (4-6) and  $\rho^+ a_+^3 = \text{const.}$  read

$$\tilde{v} = \frac{\Omega^{0.6}}{6} \left( 1 - \frac{\rho^-}{\rho^+} \right) \left\{ 1 + \left( \frac{r_i^+}{r^+} \right)^3 \right\}. \quad (4-9)$$

Figure 3 shows plots of  $\tilde{v}_0$  v.s.  $\Omega_0$  for compensated voids, where the subscript 0 denotes quantities at the present. (The details of the analysis for the homogeneous background were given by Sakai, Maeda, & Sato (1993). In the linear case (a), our numerical result is in good accordance with the result in LPT. Even in the nonlinear case (b), the difference between the two results is relatively small (up to 10%).

Let us turn to the case of uncompensated voids. Obviously the term  $(r_i^+/r^+)^3$  in equation (4-9) represents the effect of MV mass: as the comoving radius  $r$  increases, the effect of MV mass decreases. This argument explains the result that the eventual behavior of  $\tilde{v}$  does not depend on MV mass, as shown in Figure 1(b).

## 5. Effect of CMB radiation within a void

As we mentioned in the introduction, Pim & Lake (1986,1988) showed that, if we include CMB radiation, the shell expands much faster than that in the absence of radiation, and that its asymptotic behavior is  $R \propto t$  even in the flat universe. Here we re-examine the effect of radiation in the flat FRW background:  $m = 0$  and  $k^+ = 0$ .

First, let us reproduce the results of Pim & Lake (1986). As background matter, a mixture of dust ( $\rho_d^+$ ) and blackbody radiation ( $\rho_r^+$ ) is considered:

$$\rho^+ = \rho_d^+ + \rho_r^+ = \rho_{cr} \equiv \frac{3H_+^2}{8\pi G}, \quad p^+ = \frac{\rho_r^+}{3} \quad (5-1)$$

Setting the present temperature Hubble parameter as  $T_0 = 2.7\text{K}$  and  $H_0^+ = 100/\text{kms/Mpc}$  and the using the relation,

$$\rho_r^+ = \frac{8\pi^5}{15} \frac{k_B^4 T^4}{h^3 c^3}, \quad (5-2)$$

the background model is completely fixed. The interior is assumed to be the flat FRW spacetime with radiation only, of which abundance ( $\rho_r^-$ ) is characterized by a parameter,

$$\alpha \equiv \left( \frac{\rho_r^-}{\rho^+} \right)_i. \quad (5-3)$$

For matter fluid on the shell, they assume that the equation of state has a form,  $\varpi = \epsilon\sigma$ . In one of their calculations, the initial parameters are fixed as follows:

$$\begin{aligned} z_i = 1000, \quad v_i^+ = 0.1, \quad R_i H_i^+ = 0.1, \quad \epsilon = 0, \quad k_- = 0, \\ \alpha = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, \text{ or } 10^{-5}. \end{aligned} \quad (5-4)$$

Integrating the equations of motion in section 2, we obtain the result in Figure 4(a), which is the reproduction of Figure 5 of Pim & Lake (1986).

This figure tells us that, no matter how little radiation exists, it affects the shell’s motion significantly. Because this result was surprising to us, we examine their analysis. As a result, we find that their assumption of  $k^- = 0$  was inappropriate for the following reason. If  $k^\pm = 0$  and  $\rho^+ > \rho^-$ , the Friedmann equation reads  $H^+ > H^-$ . As Sato (1982) and Sato & Maeda (1983) argued, however, a thin shell is formed by compression of matter like a snow-plow mechanism when the inner expansion is faster than the background expansion, i.e.,  $H^- < H^+$ . Therefore, the assumption of  $k^\pm = 0$  is inconsistent with the thin shell description. If we still used the thin-shell equations for the case where the shell expands faster than the interior matter fluid, a part of the shell mass would be forced to “evaporate” so as to keep the inside homogeneous, i.e., the shell would emit mass and accelerate, which seems unphysical. This explains the odd behavior in Figure 4(a).

Here we re-analyze voids with CMB radiation. Although the exact value of  $H_i^-/H_i^+$  cannot be determined without knowing the formation process, the consistency with thin shell requires  $H^- \geq H^+$ . Because we still assume the background universe to be flat, the inside region should be open. Adopting  $H_i^+ = H_i^-$  instead of  $k^- = 0$  and leaving the other conditions unchanged, we solve the equation of motion. The result is reported in Figure 4(b), showing that the effect of radiation is much smaller.

We should note, however, that it is not so fruitful to investigate further details of the motion of voids including radiation in this approach. Because we do not know the physical process of radiation around the shell, the equation of state ( $\epsilon$ ) is not determined; furthermore, even the validity of the thin-shell approximation is not clear. What we can conclude is that, under the condition that the thin-shell approximation is valid, the effect of CMB radiation on void expansion is negligible.

## 6. Summary

As a model of nonlinear structure, we have considered a relativistic void in the expanding universe, and discussed peculiar velocities.

(1) In order to investigate the dynamics of shells with uncompensated mass, we have adopted McVittie spacetime as a background universe. Although the motion itself is quite different from that of a compensated void, as shown by Haines & Harris (1993), the present peculiar velocities are unaffected by MV mass.

(2) We discuss the relation between  $\Omega_0$  and peculiar velocities, comparing the results in the present model with those in the linear perturbation theory. For nonlinear voids, the quantitative difference between these two results is up to 10%, which is relatively small.

(3) Because Pim & Lake (1986,1988) arrived at the surprising conclusion that the effect of a small amount of CMB radiation is significant, we have re-examined it. We have shown that their results are due to inappropriate initial conditions. With modified initial conditions, the effect of radiation turns out to be negligible.

Although we have investigated only specific models of nonlinear structure, our results (1)-(3), as a whole, indicate that the formula for peculiar velocities in the linear perturbation theory can apply approximately to nonlinear voids.

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# FIGURES

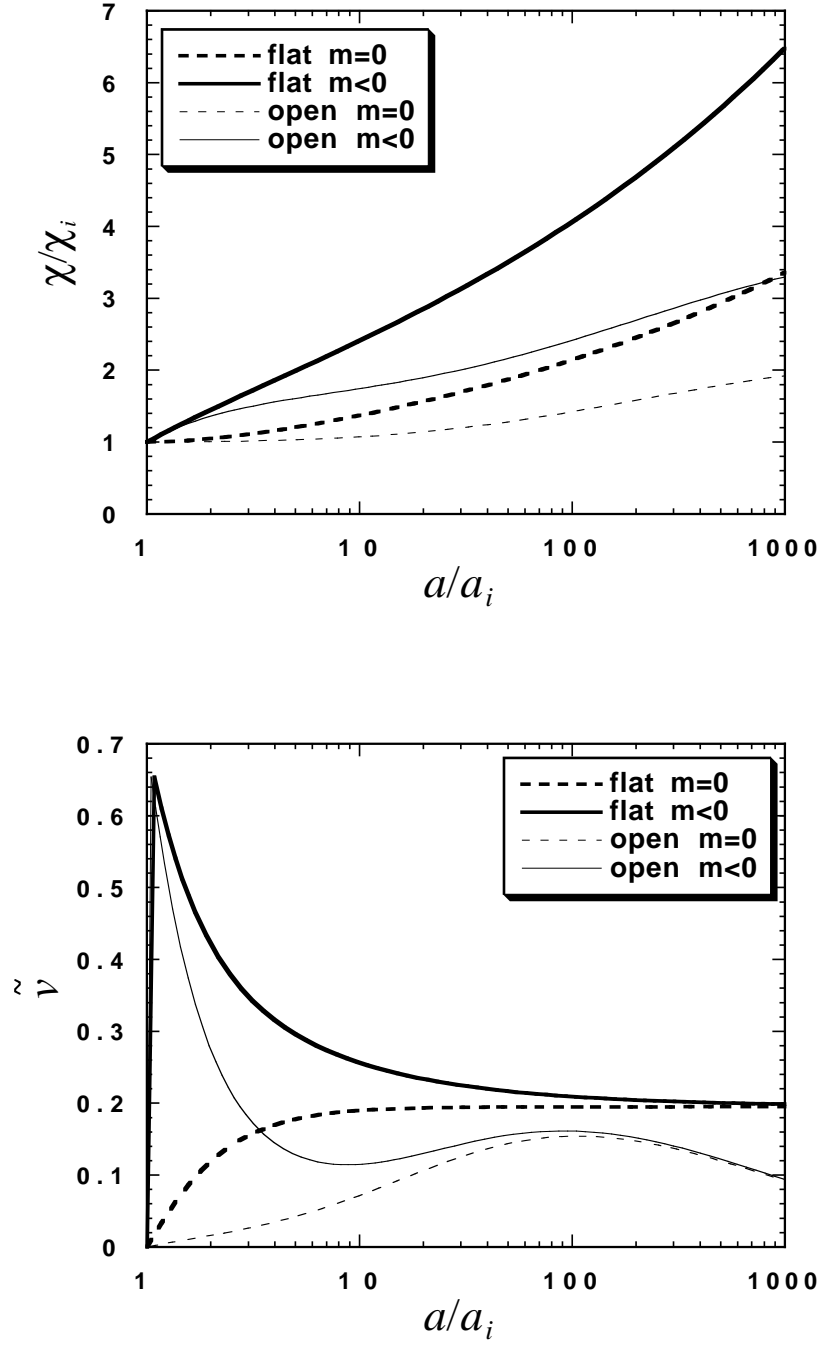


FIG. 1. Dynamics of compensated ( $m = 0$ ) and uncompensated ( $m < 0$ ) voids in flat and open ( $\Omega_0 = 0.3$ ) universes. (a) Plots of  $\chi/\chi_i$ . v.s.  $a/a_i$ . (b) Plots of  $\tilde{v}$ . v.s.  $a/a_i$ .

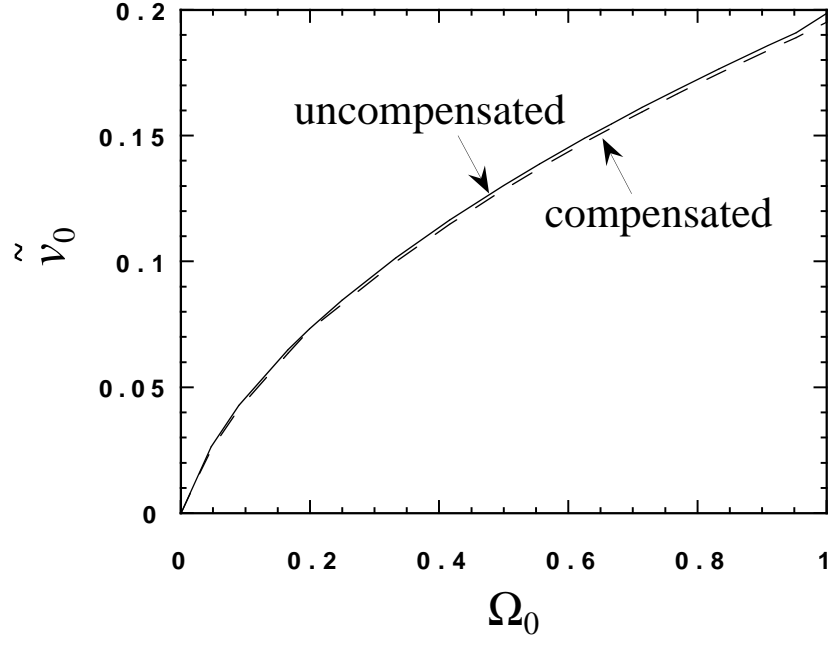


FIG. 2. Relation between  $\tilde{\nu}_0$  and  $\Omega_0$  for compensated ( $m = 0$ ) and uncompensated ( $m < 0$ ) voids.

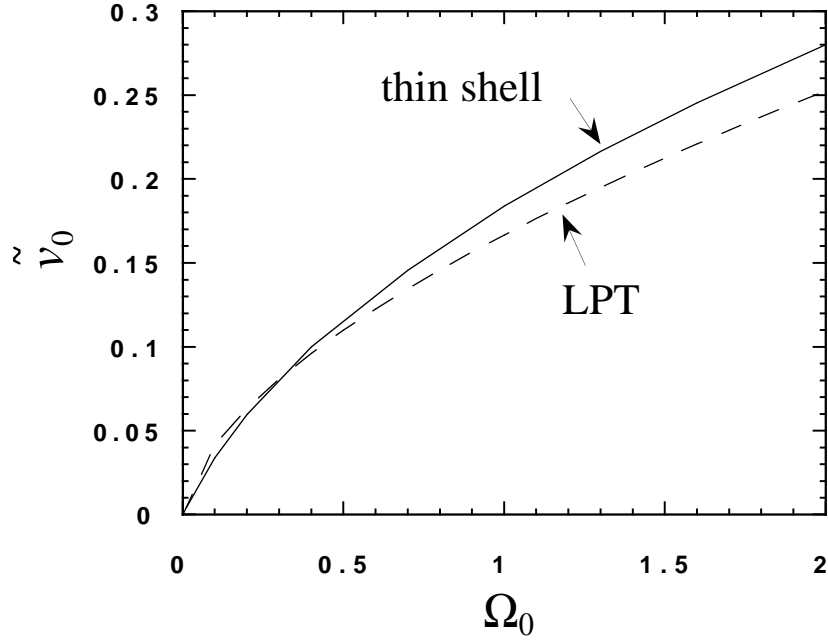
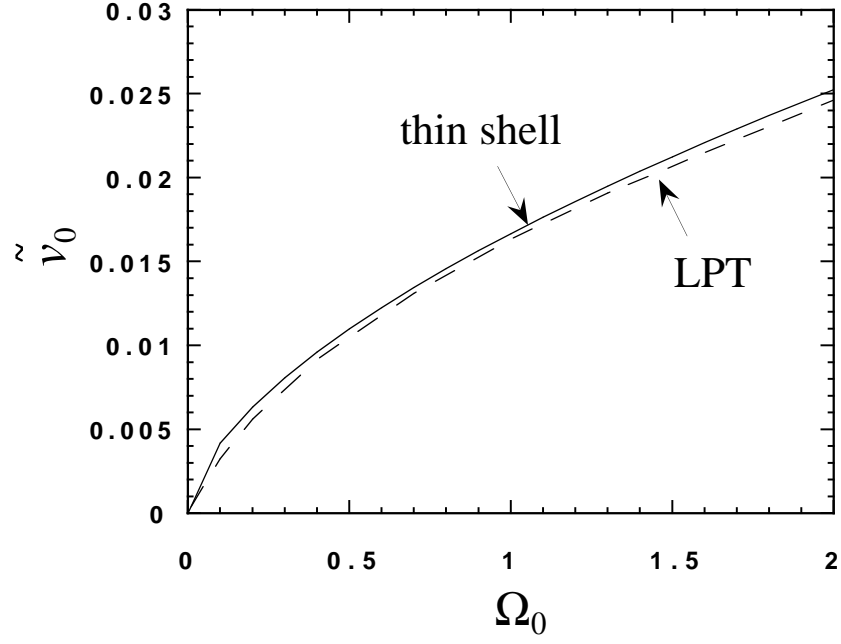


FIG. 3. Relation between  $\tilde{v}_0$  and  $\Omega_0$  in the relativistic thin-shell model and in the linear perturbation theory (LPT). (a) Linear void:  $\rho_0^-/\rho_0^+ = 0.9$ . (b) Nonlinear void:  $\rho_0^- = 0$ .



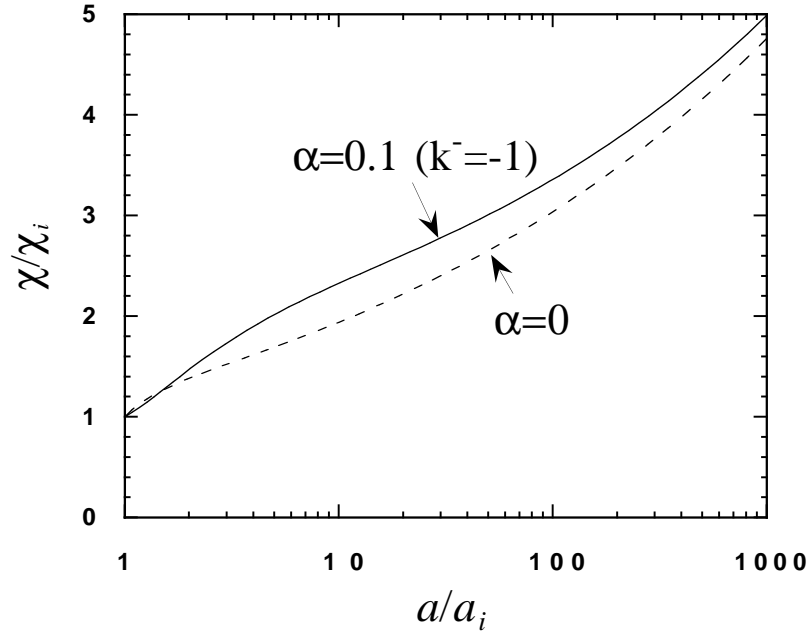
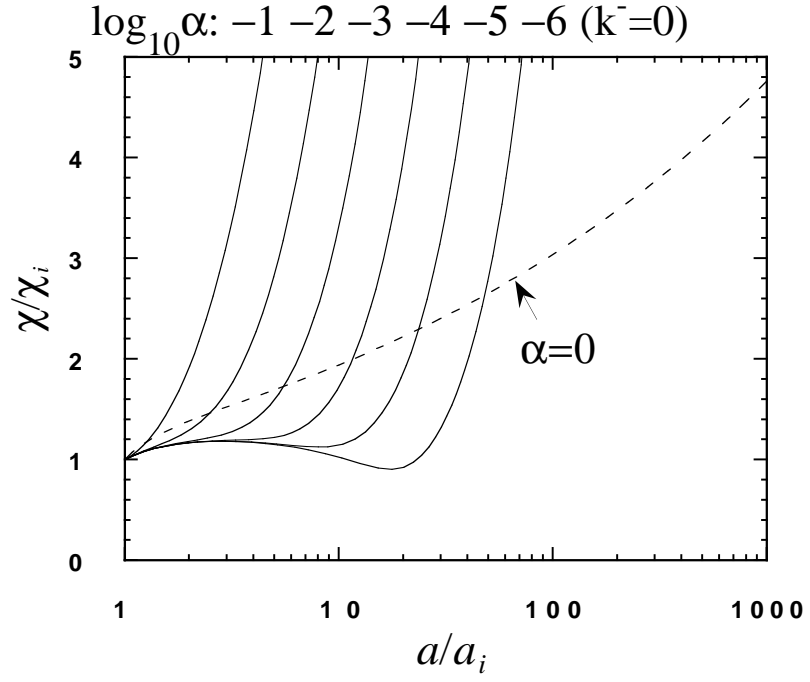


FIG. 4. Effect of CMB radiation. (a) Reproduction of Fig. 5 of Pim & Lake (1986). Both the outside and inside are assumed to be flat FRW. Results for various  $\alpha$  are presented. (b) The outside is flat, but the inside is open:  $H_i^+ = H_i^-$ ,  $\alpha = 0.1$ .