3-24-1997

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Borgani, Stefano; da Costa, Luiz N.; Freudling, Wolfram; Giovanelli, Riccardo; Haynes, Martha P.; Salzer, John; and Wegner, Gary, "Peculiar Velocities of Clusters in Cold Dark Matter Models" (1997). *Open Dartmouth: Peer-reviewed articles by Dartmouth faculty*. 2292.
https://digitalcommons.dartmouth.edu/facoa/2292

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Peculiar Velocities of Clusters in CDM Models

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Received ______________________; accepted ______________________

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[1] The National Astronomy and Ionosphere Center is operated by Cornell University under a cooperative agreement with the National Science Foundation.

ABSTRACT

Recently, peculiar velocity measurements became available for a new sample of galaxy clusters, hereafter the SCI sample. From an accurately calibrated Tully–Fisher relation for spiral galaxies, we compute the rms peculiar velocity, V_{rms}, and compare it to the linear theory predictions of COBE–normalized low–density and open CDM models (ΛCDM and OCDM, respectively). Confidence levels for model rejection are estimated using a Monte Carlo procedure to generate for each model a large ensemble of artificial data sets. Following Zaroubi et al. (1997), we express our results in terms of constraints on the (Ω_0, n_{pr}, h) parameter space. Such constraints turn into $\sigma_8\Omega_0^{0.6} = 0.50^{+0.25}_{-0.14}$ at the 90% c.l., thus in agreement with results from cluster abundance. We show that our constraints are also consistent with those implied by the shape of the galaxy power spectrum within a rather wide range for the values of the model parameters. Finally, we point out that our findings disagree at about the 3σ level with respect to those by Zaroubi et al. (1997), based on the Mark III catalogue, which tend to prefer larger Ω_0 values within the CDM class of models.

1. Introduction

Peculiar velocities of clusters have recently been used by several authors to set stringent constraints on cosmological models (e.g., Croft & Efstathiou 1994; Cen, Bahcall & Gramann 1994; Bahcall & Oh 1996; Moscardini et al. 1996). Although clusters sample the large–scale flows much more sparsely than galaxies, their peculiar velocity can be measured more accurately if distances are available for several cluster galaxies. Several of such previous analyses were, however, based on non–homogeneous compilations, with cluster velocities taken from different parent samples.

In this *Letter* we analyze the new sample of cluster peculiar velocities described by Giovanelli et al. (1997a; SCI hereafter), consisting of accurate and uniform I-band photometry and velocity width measurements for about 800 spiral galaxies in the fields of 24 clusters. Of the 24 clusters, we consider 18, pruning the six paired clusters (A2197/A2199, S805=Pavo II/Pavo, and A2634/A2666) in order to avoid possible ambiguities in membership assignment (see Giovanelli et al. 1997a).

Peculiar velocities for these clusters were determined from a a well-calibrated I-band Tully–Fisher relation constructed from all 24 clusters as described in Giovanelli et al. (1996b). An earlier version of this sample has been already analyzed by Bahcall & Oh (1996) and Moscardini et al. (1996), who compared the results with numerical simulations of several CDM–like models. Within this class of models, both analyses consistently showed that this data set favors a low–density Universe, with $0.2 \lesssim \Omega_0 \lesssim 0.4$. Furthermore, Moscardini et al. (1996) also compared the SCI sample with Hudson's (1994) compilation of cluster velocities. They found that the SCI sample provides systematically smaller velocities than those of Hudson, again suggesting a low value of Ω_0.

The analysis that we present in this *Letter* is entirely based on linear theory, the reliability of which to describe cluster motions is briefly discussed. Model predictions are worked out for purely CDM models with $\Omega_0 \leq 1$ and both flat and open geometry. Avoiding the need to resort to numerical simulations allows us to probe the model parameter space in a much more accurate way. The resulting constraints on the CDM models are also compared with those derived from the cluster abundances, the galaxy power–spectrum shape and the Mark III data as analyzed by Zaroubi et al. (1997)

2. The analysis

Our analysis is based on comparing the rms cluster velocity, V_{rms} from the SCI sample and for models. Its observational estimate for the SCI clusters gives a one dimensional $V_{rms}^{obs} = 266 \pm 30\,\mathrm{km\,s^{-1}}$, where the uncertainty represents the 1σ scatter over 10^5 Montecarlo realizations of the real sample, each one generated from an *a priori* Gaussian distribution having the same V_{rms} as the SCI data set and velocities convolved with the observational errors.

Linear gravitational instability predicts the one–dimensional rms velocity to be

$$V_{rms} = \frac{H_0 f(\Omega_0)}{\sqrt{3}} \left[\frac{1}{2\pi^2}\int_0^\infty dk\, P(k)\, W^2(kR)\right]^{1/2} , \tag{1}$$

where $f(\Omega_0) \simeq \Omega_0^{0.6}$, $P(k)$ is the model power spectrum and $W(kR)$ is the window function that specifies the “shape” and the size R of the linear density fluctuations, which generate clusters. Bahcall, Gramann & Cen (1994) and Croft & Efstathiou (1995) verified that eq.(1) provides a rather good fit to the cluster rms velocity generated by N–body simulations. By using the Gaussian window $W(kR) = \exp(-k^2R^2/2)$, we found that eq.(1) provides the best fit to the Borgani et al. (1997) N–body outputs for a variety of models by taking $R = 3.9\,h^{-1}\mathrm{Mpc}$ ($H_0 = 100h\,\mathrm{km\,s^{-1}}$ $\mathrm{Mpc^{-1}}$), corresponding to a typical cluster mass $M_{cl} \simeq 2.6 \times 10^{14}\Omega_0 h^{-1} M_\odot$. We adopt this value in the following analysis.

We take the power spectrum to be $P(k) = Ak^{n_{pr}}T^2(k)$, where

$$T(q) = \frac{\ln(1+2.34q)}{2.34q}\left[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4\right]^{-1/4} \tag{2}$$

is the CDM transfer function provided by Bardeen et al. (1986). Here $q = k/\Gamma h$ where $\Gamma = \Omega_0 h \exp(-\Omega_b - (2h)^{1/2}\Omega_b/\Omega_0)$ is the “shape” parameter, which takes into account the presence of a non–negligible baryon fraction, Ω_b (e.g., Sugiyama 1995). We take

$\Omega_b h^2 = 0.024$ (e.g., Tytler et al. 1995). The constant A is fixed by the 4–year *COBE* normalization recipes of Bunn & White (1996; see their eqs. 29 and 31) and Hu & White (1997; see their eq. 6) for flat low–density (ΛCDM) and open (OCDM) models for both vanishing and non–vanishing tensor mode contributions to CMB anisotropies. In the case of ΛCDM, Bunn & White (1996) consider the case $T/S = 7(1 - n_{pr})$ for the ratio betweem the quadrupole moments of the tensor and scalar modes in the expansion of the angular temperature fluctuations as generated by power–law inflation (e.g., Crittenden et al. 1993, and references therein). As for OCDM, the normalization for the $T/S \neq 0$ case is provided by Hu & White (1997) in the case of "minimal" tensor anisotropies. The density parameter Ω_0 and the primordial spectral index n_{pr} are varied within the ranges where the normalization fits are reliable, namely $0.2 \leq \Omega_0 \leq 1$ and $0.7 \leq n_{pr} \leq 1.2$ ($0.7 \leq n_{pr} \leq 1$) for the cases without (with) a tensor mode contribution.

The family of models we consider is specified by the three parameters (Ω_0, n_{pr}, h). Results will be presented by keeping one of them fixed, in the form of slices of the three–dimensional parameter space. Colberg et al. (1997) have recently pointed out that cluster peculiar velocities are almost independent of the density parameter, once cosmological models are normalized to reproduce the cluster abundance, according to the recipe by Eke et al. (1996). Here we follow the different approach of imposing the COBE normalization, since we regard CMB temperature anisotropies as a more stable and robust constraint than cluster abundance for a fixed choice of model parameters.

In order to establish the confidence level for the validity of a given model, we adopt the following procedure. Let v_i and σ_i be the velocity and its error for the i-th real cluster ($i = 1, \ldots, 18$). For a given model, we generate Montecarlo samples, each containing 18 velocities, V_i, drawn from a Gaussian distribution, having dispersion given by eq.(1). For each sample we convolve the i–th model velocity with the observational error σ_i and estimate the resulting rms velocity. For each sample, every cluster's velocity is newly estimated as a Gaussian deviate of the mean V_i and dispersion σ_i, and the rms velocity of the sample then computed.

For each model we generate $N = 10^4$ samples and then compute the fraction $\mathcal{F}$ of them with V_{rms}^{j} $(j = 1, \ldots, N)$ at least as discrepant as V_{rms}^{obs} with respect to their average value, $N^{-1}\sum_j V_{rms}^{j}$: the smaller the value of $\mathcal{F}$, the smaller the probability that V_{rms}^{obs} is generated by chance by that model, the larger the probability $\mathcal{P} = 1 - \mathcal{F}$ that the model itself is rejected.

Figure 1 shows the results of our analysis for scale–free (i.e., $n_{pr} = 1$) ΛCDM and OCDM models, also comparing them to other observational constraints. The contours indicate the iso–probability levels for the model exclusion. The outermost contour is for $\mathcal{P} = 90\%$ confidence level, while different levels are equi–spaced in logarithmic units by $\Delta(\log \mathcal{P}) = 0.1$. The heavily shaded area indicates the 1σ confidence level from the cluster abundance by Eke et al. (1996). By fitting the X–ray cluster temperature function with CDM model predictions, they found $\sigma_8\Omega_0^{\alpha} = 0.52 \pm 0.04$, with $\alpha = 0.52 - 0.13\Omega_0$ for ΛCDM and $\alpha = 0.46 - 0.10\Omega_0$ for OCDM (σ_8 is the linear rms density fluctuation within a top–hat sphere of $8\,h^{-1}$Mpc). [2] The medium–weight shaded area is the 95% confidence level from the fitting by Liddle et al. (1996) to the shape of the APM galaxy power spectrum by Peacock & Dodds (1994): $\Gamma = 0.23 - 0.28(1 - 1/n_{pr})$ with errors of about 16%. A consistent result has also been found by Borgani et al. (1997) from the analysis of the cluster distribution. The lightly shaded area shows the 90% confidence level by Zaroubi et al. (1997; Z97 hereafter) from the likelihood analysis of the Mark III galaxy peculiar velocities (Willick et al. 1996), whose results are reported in Table 1. The dashed curves are for different values for the age of the Universe: $t_0 = 9, 11, 13, 15, 17$ Gyrs from upper to lower curves.

Our results differ with respect to those by Z97. The difference is larger for ΛCDM models, for which the discrepancy is at $\sim 3\sigma$ level (note that the corresponding 90% confidence level are at most marginally overlapping), the latter favoring larger Ω_0 values

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[2] Pen (1996) recently pointed out that the scaling by Eke et al. (1996) somewhat underestimates the value of σ_8 at small Ω_0 values. In particular, he claimed that for a ΛCDM model with $\Omega_0 \simeq 0.35$ σ_8 should be $\sim 17\%$ larger. We checked that the central value for the h interval corresponding to such an Ω_0 increase from $h = 0.66$ to $h = 0.74$.

(at a fixed h). This result points in the same direction as that found by Moscardini et al. (1996). On the other hand, the constraints we set on CDM models are quite consistent with those coming from the $P(k)$ shape and the cluster abundance, in a rather broad range of Ω_0 and h values. For instance, if we impose ages in the range $13 \lesssim t_0 \lesssim 15$ Gyrs, ΛCDM models require $0.35 \lesssim \Omega_0 \lesssim 0.50$ with $0.50 \lesssim h \lesssim 0.65$, while OCDM models require $0.50 \lesssim \Omega_0 \lesssim 0.70$ with $0.45 \lesssim h \lesssim 0.60$. On the contrary, the results by Z97 are rather discrepant with both such constraints, especially with the cluster abundance.

A similar picture also emerges as tilted (i.e., $n_{pr} \neq 1$) models are considered. Figure 2 is analogous to Fig.1 , but with results plotted in the n_{pr}–Ω_0 plane, taking $h = 0.65$ and 0.55 for ΛCDM and OCDM models, respectively. For both classes of models, taking $T/S \neq 0$ has the effect of *(a)* narrowing the permitted region in the parameter space and *(b)* decreasing the need for a tilt (cf. also Z97). Tilting $P(k)$ breaks the degeneracy of the $P(k)$ shape with the other constraints. Fixing $h = 0.65$ (e.g. Giovanelli et al. 1997c) and $t_0 \simeq 13$ Gyrs for ΛCDM would require $\Omega_0 = 0.43$; this turns into $0.85 \lesssim n_{pr} \lesssim 0.95$ and $0.90 \lesssim n_{pr} \lesssim 0.96$ for $T/S = 0$ and $T/S = 7(1 - n_{pr})$, respectively. Consistency between the $P(k)$ shape and Z97 are attained for $n_{pr} \gtrsim 1$ and $\Omega_0 \lesssim 0.5$, while the cluster abundance is still largely missed. As for OCDM models, taking $h = 0.55$ and $t_0 \simeq 13$ Gyrs implies $\Omega_0 \simeq 0.65$ and $0.84 \lesssim n_{pr} \lesssim 0.94$ ($0.84 \lesssim n_{pr} \lesssim 0.94$) for $T/S = 0$ ($\neq 0$). A substantially larger h value would turn into too small Ω_0 values, unless $t_0 < 13$ Gyrs. Again, the SCI cluster velocities are consistent in all the cases with the other two constraints for reasonable values of the model parameters.

In order to better quantify the difference with respect to the constraints provided by the Z97 analysis, we fit the same combination of parameters, $\Omega_0 h_{50}^{\mu} n_{pr}^{\nu} = C$ ($h_{50} = 2\,h$: Hubble constant in units of $50\,\mathrm{km\,s^{-1}Mpc^{-1}}$), considered in that paper and the results are in Table 1. Although the shape of the relation (i.e., the values of μ and ν) is quite similar, its amplitude C is significantly different. This confirms that, for fixed h and n_{pr} values, our results favor a lower density parameter. We have also computed the best fit to the quantity $\sigma_8 \Omega_0^{0.6}$, which fixes the amplitude of the velocity field in linear theory. We find $\sigma_8 \Omega_0^{0.6} = 0.50^{+0.25}_{-0.17}$ (errors correspond to 90% confidence level), which again agrees with the constraints from the cluster abundance, and is significantly smaller than $\sigma_8 \Omega_0^{0.6} = 0.88 \pm 0.15$

that derived by Z97. Willick et al. (1996) recently compared the Mark III data with velocity and density fields reconstructed from the 1.2 Jy *IRAS* survey. Quite interesetingly, they obtained $\sigma_8\Omega_0^{0.6} = 0.34 \pm 0.05$ (cf. their Fig. 20), thus at variance with respect to the results by Z96, also based on the Mark III sample, and rather consistent with our results.

3. Conclusions

We have performed a detailed comparison of the cluster peculiar velocities in the SCI catalog with those predicted by COBE–normalized CDM models, using linear theory. This comparison has been made by computing the rms cluster velocity, V_{rms}, for data and models, and estimating the likelihood that the observed value $V_{rms} = 266 \pm 30\,\mathrm{km\,s^{-1}}$ is consistent with a given model.

Confidence levels for rejecting models were determined using a Monte Carlo procedure which generates a large number (10^4) of mock samples from each model. The main goal of our analysis has been to impose constraints on the space of (Ω_0, n_{pr}, h) parameters for CDM models. We have compared our results with those of Z97, and with the constraints that have been established from the properties of clustering of galaxies as expressed by the shape of the power–spectrum and recent determinations of cluster abundance.

Our results can be summarized as follows:

(a) Velocities of SCI clusters point toward a low–normalization model, characterized by $\sigma_8\Omega_0^{0.6} = 0.50^{+0.25}_{-0.17}$. This result agrees with the independent constraint coming from the abundance of galaxy clusters.

(b) Our results disagree at about the 3σ level with those of Z97, based on Mark III, the latter generally indicating higher velocities and, therefore, favoring larger Ω_0 values for fixed h and n_{pr} parameters (cf. Table 1). On the other hand, we are quite consistent with the analysis by Willick et al. (1996), which is also based on the Mark III sample.

(c) The results agree well with those from the analysis of field spirals in the new SFI sample (da Costa et al. 1997; Freudling et al. 1997).

The conclusions that we draw in this *Letter* about the values of the model parameters strictly hold, only for the CDM class of models. For instance, Cold+Hot DM models, are characterized by different power spectrum shapes and smaller COBE–normalized σ_8 values for a fixed choice of (Ω_0, n_{pr}, h), depending on the amount and the nature of the hot component (e.g., Primack 1996, and references therein). We postpone to a forthcoming paper the analysis of a wider class of cosmological models, as well as the comparison with other data sets for galaxy and cluster peculiar velocities.

Acknowledgments

SB wishes to acknowledge ESO in Garching and SISSA in Trieste for their hospitality during the preparation of this work. This work was supported by NSF grants AST94-20505 to RG, AST90-14860 and AST9023450 to MH and AST93-47714 to GW who also received partial support from ESO in Garching.

REFERENCES

Bahcall N.A., Gramann M., Cen R., 1994, ApJ, 436, 23

Bahcall N.A., Oh S.P., 1996, ApJ, 462, L49

Bardeen J.M., Bond J.R., Kaiser N., Szalay A.S., 1986, 304, 15

Borgani S., Moscardini L., Plionis M., Górski K.M., Holtzman J., Klypin A., Primack J.R., Smith C.C., Stompor R., 1997, NewA, 1, 321

Borgani S., Gardini A., Girardi M., Gottlöber S., 1997, NewA, in press, preprint astro–ph/9702154

Bunn E.F., White M., 1996, ApJ, in press, preprint astro–ph/9607060

Cen R., Bahcall N.A., Gramann M., 1994, ApJ, 437, L51

Colberg J.M., White S.D.M., Jenkins A., Pearce F.R., Frenk C.S., Thomas P.A., Hutchings R., Couchman H.M.P., Peacock J.A., Efstathiou G.P., Nelson A.H., 1997, in Proceedings of the Ringberg Workshop on "Large Scale Structure", ed D. Hamilton, preprint astro–ph/9702086

Crittenden R., Bond J.R., Davis R.L., Efstathiou G., Steinhardt P.J., 1993, Phys. Rev. Lett., 71, 324

Croft R.A.C., Efstathiou G., 1995, MNRAS, 268, L23

da Costa, L. N., et al., 1997, in preparation

Eke V.R., Cole S., Frenk C.S., 1996, MNRAS, 282, 263

Freudling, W., et al. 1997, in preparation

Giovanelli R., Haynes M.P., Herter T., Vogt N.P., Wegner G., Salzer J.J., da Costa L.N., Freudling W., 1997a, AJ, 113, 22

Giovanelli R., Haynes M.P., Herter T., Vogt N.P., da Costa L.N., Freudling W., Wegner G., Salzer J.J., 1997b, AJ, 113, 53

Giovanelli, R., Haynes, M. P., da Costa, L. N., Freudling, W., Salzer, J. J., Wegner, G., 1997c, ApJ, 477, L1

Hu W., White M., 1997, ApJ Lett, submitted, preprint astro–ph/9701210

Hudson M.J., 1994, MNRAS, 266, 475

Liddle A.R., Lyth D.H., Schaefer R.K., Shafi Q., Viana P.T.P., 1996, MNRAS, 281, 531

Moscardini L., Branchini E., Tini Brunozzi P., Borgani S., Plionis M., Coles P., 1996, MNRAS, 282, 384

Peacock J.A., Dodds S.J., 1994, MNRAS, 267, 1020

Pen U.-L., 1996, ApJ, submitted, preprint astro–ph/9610147

Primack J.R, 1996, in *Critical Dialogues in Cosmology*, ed. Neil Turok (World Scientific, in press), preprint astro–ph/9610078

Sugiyama N., 1995, ApJS, 100, 281

Tytler D., Fan X.-M., Burles S., 1995, Nature, 381, 207

Willick J.A., Courteau S., Faber S.M., Burstein D., Dekel A., Strauss M.A., 1996, ApJS, in press, preprint astro–ph/9610202

Willick J.A., Strauss M.A., Dekel A., Kolatt T., 1996, ApJ, submitted, preprint astro–ph/9612240

Zaroubi S., Zehavi I., Dekel A., Hoffman Y., Kolatt T., 1997, ApJ, in press, preprint astro–ph/9610226 (Z97)

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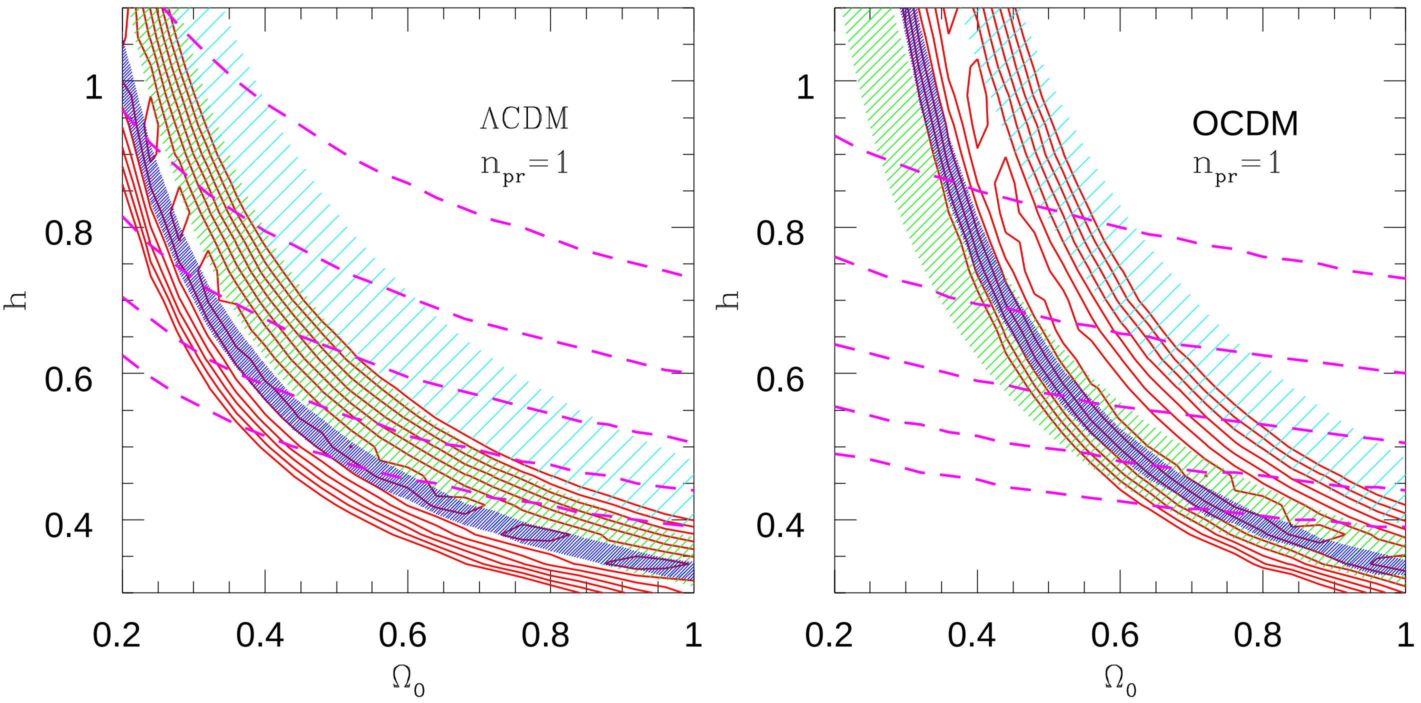
Figure captions

Figure 1. Constraints from the SC cluster sample on the (h, Ω_0) plane for scale–free $(n_{pr} = 1)$ models. The countours are the levels at equal probability $\mathcal{P}$ for model rejection. The most external level corresponds to $\mathcal{P} = 90\%$ and the spacing corresponds to $\Delta(\log \mathcal{P}) = 0.2$. The heavely shaded area is the constraint from cluster abundance (after Eke et al. 1996), the medium–weight shaded area is for the shape of the APM galaxy power spectrum (after Viana et al. 1996) and the lightly shaded area is from the analysis of Mark III velocities by Zaroubi et al. (1996). Dashed curves indicate different ages for the Universe: $t_0 = 9, 11, 13, 15, 17$ from upper to lower curves.

Figure 2. It is analogous to Fig. 1, but on the (n_{pr}, Ω_0) plane. Different ages of the Universe are now indicated with the vertical dashed lines.

Table 1: Values of thw fitting parameters for the relation $\Omega_0 h_{50}^{\mu} n_{pr}^{\nu} = C$ ($h_{50} = 2\,h$: Hubble constant in units of $50\,\mathrm{km\,s^{-1}Mpc^{-1}}$), from our analysis and from that by Zaroubi et al. (1997; Z97).

Model	This paper			Z97		
	μ	ν	C	μ	ν	C
ΛCDM $T/S = 0$	1.30	$1.8^{+0.2}_{-0.4}$	$0.53^{+0.17}_{-0.14}$	1.30	2.0	0.83 ± 0.12
ΛCDM $T/S = 7(1 - n_{pr})$	1.30	$3.2^{+0.4}_{-0.7}$	$0.53^{+0.17}_{-0.14}$	0.87	3.4	0.83 ± 0.12
OCDM $T/S = 0$	0.87	$1.3^{+0.1}_{-0.2}$	$0.67^{+0.15}_{-0.14}$	0.95	1.4	0.88 ± 0.09
OCDM $T/S \neq 0$	0.87	$2.2^{+0.2}_{-0.3}$	$0.67^{+0.15}_{-0.14}$			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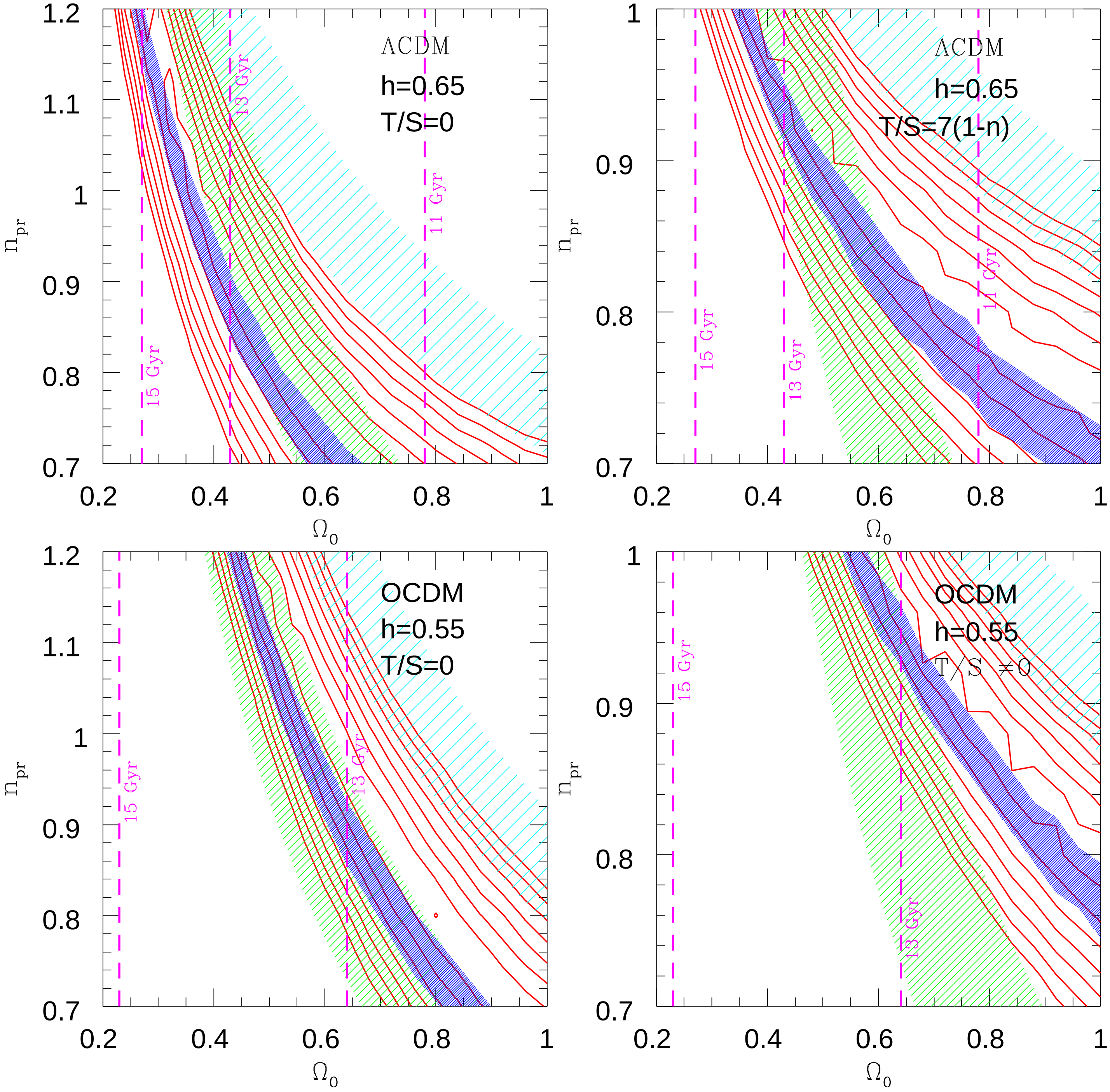
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13 Gyr
11 Gyr
n_{pr}
Ω_0
1.2
1.1
1
0.9
0.8
0.7
0.2
0.4
0.6
0.8
1
ΛCDM
h=0.65
T/S=7(1-n)
15 Gyr
13 Gyr
11 Gyr
n_{pr}
Ω_0
1
0.9
0.8
0.7
0.2
0.4
0.6
0.8
1
OCDM
h=0.55
T/S=0
15 Gyr
13 Gyr
n_{pr}
Ω_0
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1.1
1
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T/S ≠0
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