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A NOTCH-STRENGTHENING EFFECT IN FRESH-WATER ICE

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ABSTRACT. Tensile tests have been performed on notched and unnotched cylindrical samples of randomly oriented polycrystalline ice of controlled grain-size (between 2.2 and 7.3 mm) at a loading rate of 100 Pa s^{-1} and at a temperature of $-10\,^{\circ}$C. In the notched samples, the notch-root diameter was 80% of the base diameter. A notch-strengthening effect was observed in the large-grained ice, with fracture stresses being up to 50% higher than that for unnotched samples of the same grain-size. This notch-strengthening effect diminished as grain-size decreased, disappearing at a grain-size of ≈3 mm.

The notch-strengthening effect is explained in terms of the triaxial stress constraint at the notch root. This triaxial constraint results in a change in the controlling mechanism of fracture from crack propagation in the unnotched samples to crack initiation in the notched samples.

INTRODUCTION

Nixon and Schulson (1986) showed that, at $-10\,^{\circ}$C, the fracture toughness (K_c) of fresh-water ice increased as the loading rate ($\dot{K}$) decreased below 10 kPa m$^{1/2}$ s^{-1}. Above this loading rate the toughness was constant. They explained this transition by considering the size of the creep zone at the crack tip. By analogy with the American Society for Testing and Materials (1981) fracture-toughness testing code for metals, they defined the transition loading rate ($\dot{K}_t$) at which plane-strain conditions were violated as that loading rate at which the size of the crack-tip creep zone (Riedel and Rice, 1980) exceeded 1/50th of the notch depth (N.B. in a different sample geometry, another sample dimension may be critical in this regard). The value of $\dot{K}_t$ thus calculated was in good agreement with the observed behavior of K_c with loading rate (Nixon and Schulson, 1986, 1987). Ralston (personal communication) suggested that at the lowest loading rate ($\dot{K}$ = 0.01 kPa m$^{1/2}$ s^{-1} << $\dot{K}_t$) a notch-strengthening process may be occurring. Notch strengthening implies that the nominal section fracture stress across a notched sample is greater than that across an unnotched sample loaded under similar conditions (i.e. $\sigma_{nn} > \sigma_{un}$; see Fig. 1).

Notch strengthening is well documented for metals and normally arises because the triaxial stress at the notch tip suppresses yield until the axial stress across the notch is considerably greater than the yield stress, σ_y. From slip-line field theory, a value of $3\sigma_y$ is generally taken as the maximum possible notch yield stress, though Orowan (1945) has shown that the maximum value of this "plastic constraint factor" is 2.57. However, if the notch stress, while still too low to cause yielding in the notch, exceeds the brittle-fracture stress (σ_f) of the material, then brittle fracture may occur. In such cases a material is termed notch brittle. In steel one finds that above a certain temperature the material is notch tough because both σ_y and $3\sigma_y$ (the unnotched and notched yield stresses) are less than σ_f. As the temperature decreases below this temperature, the steel becomes notch brittle (that is σ_y is less than σ_f, but $3\sigma_y$ is greater than σ_f) until, at some still lower temperature, it is fully brittle.

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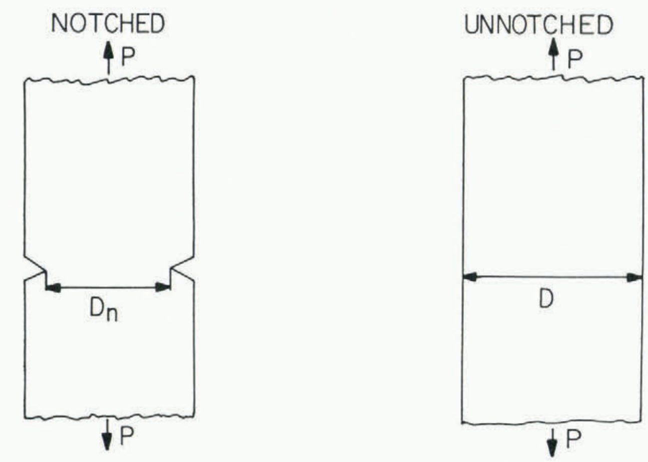

$$\sigma_{nn} = \frac{4P}{\pi D_n^2} \qquad \sigma_{un} = \frac{4P}{\pi D^2}$$

Fig. 1. Notched and unnotched samples.

The presence or otherwise of a notch-strengthening effect in ice is of more than academic interest. It is often noted that sea ice has many defects (e.g. cracks, polynas, pores, etc.) and the implication is drawn that the defected ice should be weaker than unflawed laboratory ice. Recent work (Schulson and others, 1989) shows that short cracks have no effect on the tensile strength, and that even long cracks which have blunted appear not to weaken the material. The effect of existing cracks on the strength of ice is thus far from clear. A study of notch-strengthening effects may help to clarify this complex problem. The purpose of this paper is to describe experiments designed to investigate the notch-strengthening effect in ice and to explain the results obtained in a micromechanical context.

EXPERIMENTAL PROCEDURE

Cylindrical samples of randomly oriented polycrystalline fresh-water ice were made using the procedure described by Lee and others (1984). Specimens made in this manner had an average density of 916 kg m^{-3} (i.e. 0.11% porosity) at $-5\,^{\circ}$C. Melt-water conductivity of the ice was 8.8×10^{-6} mho/cm, also at $-5\,^{\circ}$C. The ice appeared to be very clear. What few bubbles were found tended to be along the central axis of the sample, as might be expected from the freezing process. Because it has been shown that both the fracture toughness (Nixon and Schulson, 1988) and the tensile strength (Lee and Schulson, 1988) of fresh-water ice are grain-size dependent, tests were performed on samples with average grain-size (as measured by the linear intercept method) between 7.3 and 2.2 mm. Tensile tests were performed on a closed loop servo-hydraulic testing machine placed in a cold room with a temperature of $-10.0 \pm 0.2\,^{\circ}$C. All tests were performed in load control adjusted so that the stress rate in the unnotched samples, $\dot{\sigma}_{un}$, was equal to

the nominal notch stress rate in the notched samples ($\dot{\sigma}_{nn} = \sigma_{un}$ = 100 Pa s^{-1}). It should be noted that tests were not run in strain control (as is normal practice) because strain, and thus strain-rate, varies significantly with position in the notched samples. The notched and unnotched samples had an initial diameter of 91 mm. In the notched samples, this was reduced at the notch to 72 mm. The notch half-angle was 22° (see Fig. 1). Sample lengths between end caps were 254 ± 3.2 mm, and the notches were placed within 13.2 mm of the sample midpoint. Samples were notched to within 0.25 mm of their final depth on a lathe in a cold room at −14°C with a shaped notching tool. These blunt notches were then sharpened to their final depth with a razor blade mounted in a special holder for use on the tool post of the lathe. Tests on notched samples were performed between 12 and 16 h after notching. Prior to testing, samples were stored in insulated boxes at the test temperature. Since tests lasted typically 3 h, all samples were wrapped in plastic "clingfilm" to avoid sublimation during testing. Strain was measured by means of two extensometers, as used by Lee and others (1984).

RESULTS

Test results are presented in Table I. For the most part there is little scatter in the results, the exception being the

TABLE I. NOTCHED AND UNNOTCHED TENSILE STRENGTHS OF FRESH-WATER ICE

T = −10°C; $\dot{\sigma}$ = 100 Pa s^{-1}

Grain-size	*Notched?*	*Failure stress*	*Failure time*
mm		MPa	s × 10^3
7.3	yes	0.92	9.23
7.2	yes	0.91	9.12
7.3	yes	0.88	8.79
7.4	yes	0.88	8.81
7.4	no	0.60	6.03
7.2	no	0.70	7.04
5.5	yes	0.94	9.44
5.1	yes	0.96	9.57
4.0	yes	0.93	9.29
3.6	no	0.88	8.81
3.5	yes	0.84	8.43
3.6	yes	0.89	8.92
3.4	yes	0.75	7.53
3.5	yes	1.09	10.90
3.6	no	0.80	8.03
3.4	no	0.87	8.68
2.2	yes	1.03	10.30
2.1	yes	1.06	10.60
2.3	no	0.89	8.91
2.2	no	1.03	10.30

strengths of the notched samples of 3.5 mm grain-size. The unnotched samples showed increasing non-linear load-extension behavior as the grain-size decreased. The larger-grained (d > 3 mm) notched samples showed no non-linear behavior, while for the notched samples with grain-size less than 3 mm some ductility was evident. Typical load-extension curves for the tests are shown in Figure 2. Figure 3 shows the failure loads for notched and unnotched specimens as a function of grain-size. From this it is clear that a greater load is required to break an unnotched specimen than a notched specimen. However, when failure *stress*, rather than failure *load*, is considered (see Fig. 4), a different trend emerges. It can be seen that there is a definite notch-strengthening effect at the larger grain-sizes and that it diminishes as the grain-size decreases and becomes negligible for the finest-grained aggregates. A significant result, discussed below, is that while all unnotched samples had, after failure, other cracks present away from the fracture path, none of the notched samples had any remnant cracks.

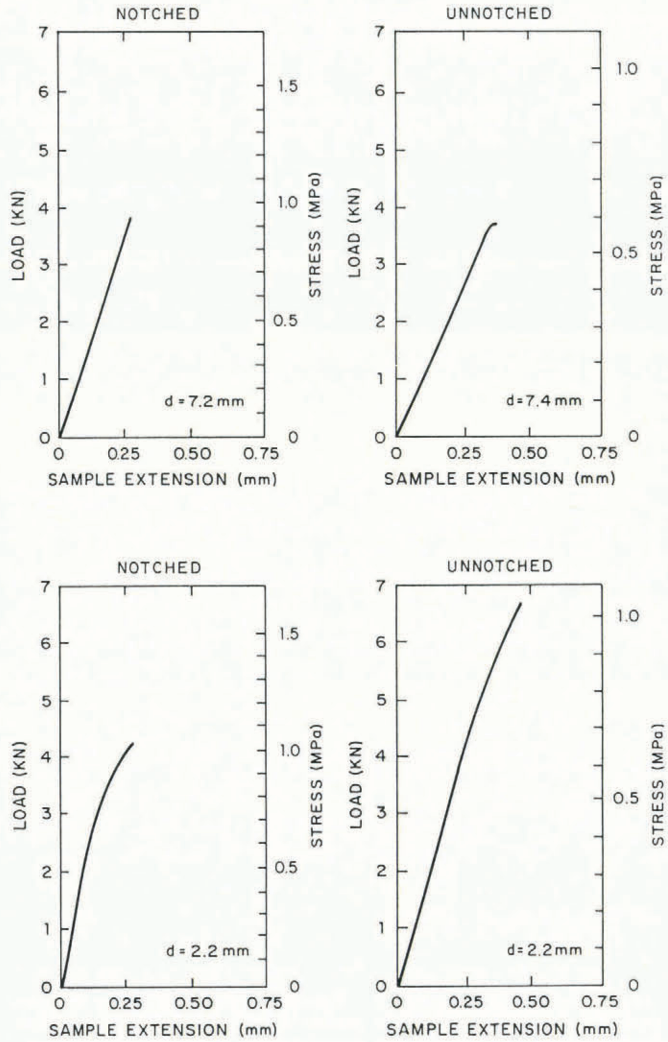

Fig. 2. Load–extension curves: (a) large-grained, notched; (b) small-grained, notched; (c) large-grained, unnotched; (d) small-grained, unnotched.

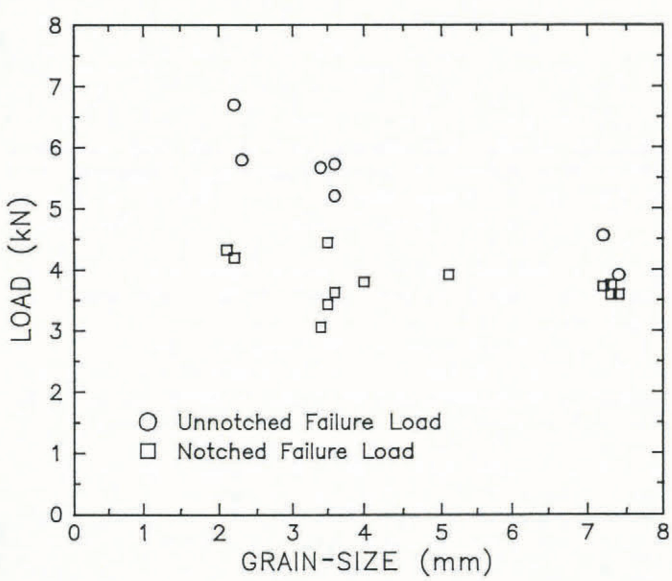

Fig. 3. Failure load vs grain-size.

DISCUSSION

It should be noted that ductility in the samples for this study was less than in samples tested under strain-rate control by Lee and Schulson (1988) at similar rates of loading. The reason for the reduced sample extension at failure in these tests, as compared with those of Lee and

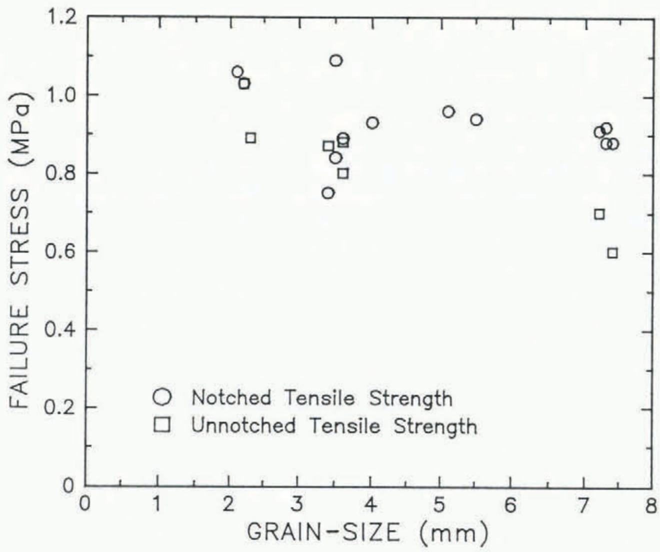

Fig. 4. Failure stress vs grain-size.

Schulson (1988), lies in the fact that the tests herein were performed under "load control". By using the load feed-back control (the load was increased at a constant rate; see above), the rate of straining increases within the sample as soon as non-linear load-extension behavior begins, which promotes earlier fracture than in constant strain-rate tests. Thus, while the fracture stress is similar to those of Lee and Schulson, the fracture strains are somewhat less (Figure 5 shows a stress–strain curve from Lee and Schulson's work for comparison with Figure 2).

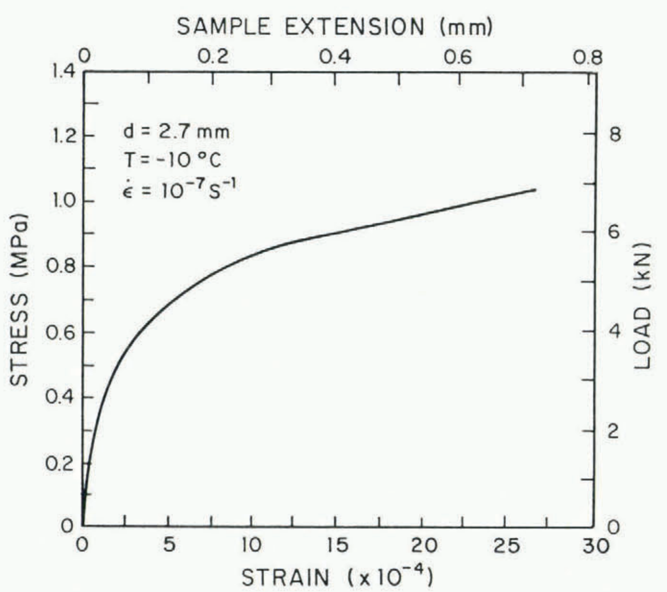

Fig. 5. Stress–strain curve from constant strain-rate test (Lee and Schulson, 1988).

As noted by Schulson and others (1984), in low strain-rate tension tests on ice ($\dot{\epsilon} = 10^{-6}\,s^{-1}$), the onset of non-linear stress–strain behavior is linked with the initiation of cracks. Under such strain-rates, cracks initiate and further stress must be applied for those cracks to propagate. The failure mode in such low strain-rate tests can be clearly identified as one of propagation of pre-existing cracks. Two observations support this. First, in this study and others (Schulson and others, 1984; Lee, unpublished), remnant cracks were observed in samples tested at low rates, clearly indicating that cracks had initiated prior to final failure. Secondly, in a number of individual tests, cracks observed prior to failure lay on the actual failure surface. These two sets of observations, further supported by theoretical discussion (Schulson and others, 1984), appear to indicate conclusively that at low strain-rates failure in tension is by crack propagation rather than crack initiation. However, at higher strain-rates ($\dot{\epsilon} = 10^{-3}\,s^{-1}$), again under tension, the stress to cause crack initiation is greater than that to cause the initiated cracks to propagate, and thus fracture occurs immediately upon crack initiation. The process of crack initiation was presumably one of microplasticity (Schulson and others, 1984; Lee and Schulson, 1988), possibly involving the piling up of dislocations at grain boundaries.

In the vicinity of a notch, such plastic flow will be suppressed because of the triaxial stress state there. In terms of principal stresses, σ_1, σ_2, σ_3 (with $\sigma_1 > \sigma_2 > \sigma_3$), as σ_2 and σ_3 become non-zero and positive σ_1 must increase to a higher value than in the unnotched case for plastic flow to occur. Of course, if the notch is sharp enough to act as a crack then initiation is not needed. However, in low-rate tests, because of crack-tip creep and the test time involved, we may expect significant crack-tip blunting. In fact, a simple calculation (see Appendix) of the crack-tip creep-zone size (Riedel and Rice, 1980) shows that for the notched sample the creep zone develops fully across the sample during the tests. Hence, if notch strengthening does occur, we would expect to find it most prevalent at low rates and would observe two effects. First, the nominal fracture stress of notched samples will be greater than those of unnotched samples. Secondly, if plotted against (grain-size)$^{-1/2}$, the notched ice strengths should have a typical Hall–Petch dependence on grain-size:

$$\sigma_{nn} = \sigma_0 + kd^{-1/2} \tag{1}$$

where σ_0 and k are constants, in constrast with the unnotched low-rate strengths (cf. Lee and Schulson, 1988) which are expressed as:

$$\sigma_{un} = Kd^{-1/2} \tag{2}$$

where K is also a constant directly related to the fracture toughness of the ice. Further, we would expect the values of σ_{nn} at a given grain-size to be between two and three times the value of the stress at which crack nucleation commences (causing non-linear stress–strain behavior) in an unnotched sample of the same grain-size loaded under similar conditions.

Figure 6 shows the test results plotted against (grain-size)$^{-1/2}$. Also shown are the tensile test results obtained by Lee and Schulson (1988) for fresh-water ice strained at a strain-rate of $10^{-7}\,s^{-1}$ at a temperature of $-10\,^{\circ}C$. As can be seen, the unnotched samples show good agreement with the results of Lee and Schulson, as would be expected. However, the notched samples exhibit higher nominal strengths at large grain-sizes. The line of best fit for the notched samples is shown (correlation coefficient, r^2

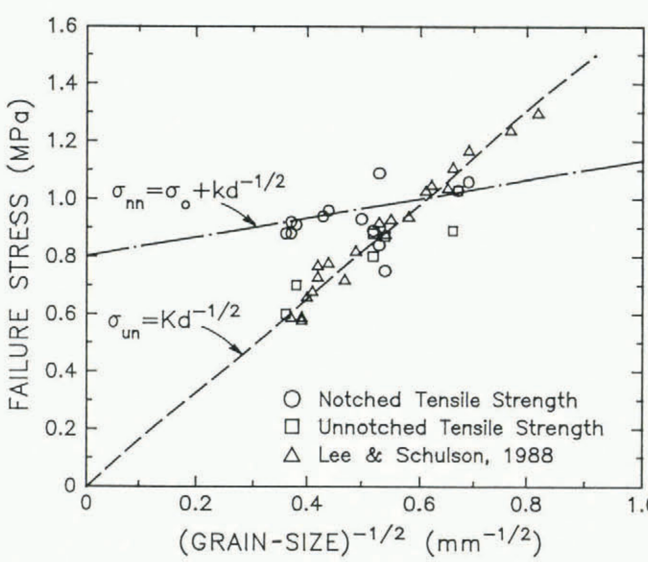

Fig. 6. Failure stress vs (grain-size)$^{-1/2}$.

= 0.82) and it has a large non-zero intercept giving values for Equation (1) of σ_0 = 0.80 MPa and k = 0.33 MPa mm$^{\frac{1}{2}}$. It is possible that the trends of strength with grain-size observed herein result from samples having too few grains across a diameter to exhibit true polycrystalline behavior. The question of how many grains are required across a sample diameter for a "good" test is one generating much debate at present, and no unambiguous and generally applicable answer has yet been obtained. In this study, it is not felt that the sample diameter/grain-size ratio is too small. Whilst for large-grained specimens this ratio is ~10 for the notched and ~13 for the unnotched samples, for the fine-grained specimens the ratios are ~33 and ~41, respectively. Assuming the fine-grained samples have a sufficiently high specimen size/grain-size ratio, it seems unlikely that the trends observed in fine-grained samples consistent with Equations (1) and (2) would continue in the large-grained samples as a result of too few grains being present in the sample. Further, if we compare the notched strengths with the stress for a 0.005% strain offset obtained from Lee (unpublished), we find (see Fig. 7) that the notched strengths are about two times greater than these offset strengths. The use of a 0.005% offset can be justifed because the initiation of the first cracks coincides with the onset of non-linear behavior on the stress–strain curve, and

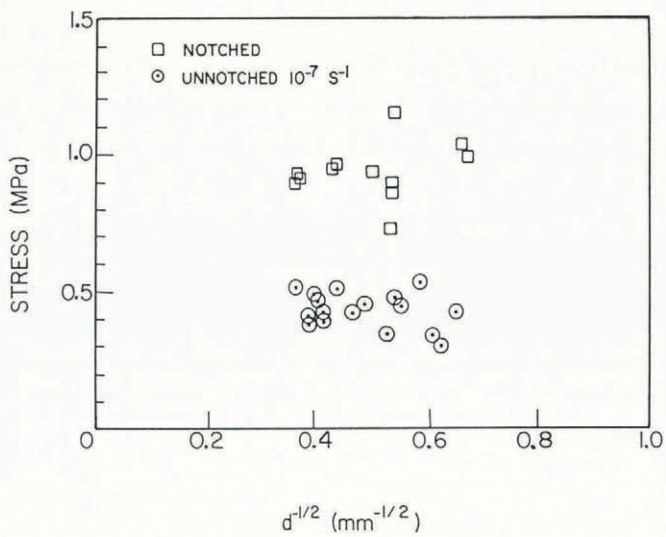

Fig. 7. Failure stress (notched) and unnotched "yield stress" (from Lee, unpublished) vs (grain-size)$^{-1/2}$.

0.005% strain gives the smallest discernible offset. If the failure of the notched samples were controlled by initiation of cracks from the notch root, we would expect the initiation to occur when the stress state local to the notch was suitable for microplasticity to occur (assuming, as noted above, that this is the crack-initiation mechanism). The shape of the notch causes a triaxial stress state at the notch which inhibits plastic flow and thus the stress to cause plastic flow in the notch region would be two to three times as high as in an unnotched sample. The fact that the notched strengths are approximately twice the offset stresses measured by Lee (unpublished) appears to provide qualitative support for the notion that the notch-strengthening effect observed in these tests is indeed due to notch constraint inhibiting initiation at the blunted notch tip. Thus fracture was controlled by the initiation of a crack from the notch root, which immediately propagated. In constrast, in the unnotched tests a number of cracks was initiated before the stress increased to the point at which one of those cracks propagated. This model is supported by the observation that there were no remnant cracks observed in the broken notched samples, a result which contrasts with the remnant cracks (up to 20 in a sample) found both in the broken unnotched samples in these tests and in the low-rate tension tests performed by Lee and Schulson (1988).

CONCLUSIONS

The following conclusions may be drawn:

(1) At low loading rates ($\dot{\sigma}$ = 100 Pa s^{-1}) and large grain-sizes ($d \geq 5$ mm), polycrystalline fresh-water ice does exhibit notch-strengthening behavior (at $-10\,^{\circ}$C).

(2) This notch-strengthening effect diminishes and disappears as the grain-size is decreased below 3 mm.

(3) The notch-strengthening effect is consistent with a model in which the presence of the notch inhibits crack initiation and causes fracture to be controlled by the stress at which a crack initiates at the notch root, while for unnotched samples the fracture stress is that to cause one among several previously initiated cracks to propagate.

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APPENDIX

Riedel and Rice (1980) gave an expression for the maximum size of the crack-tip creep zone, which, taking the value of the creep exponent n from the power-law creep equation:

$$\dot{\varepsilon} = B\sigma^n \tag{A1}$$

to be $n = 3$ (Barnes and others, 1971), gives the value of the creep-zone radius, r_{cr} to be:

$$r_{cr} = \frac{(K_c)^3}{2\pi} \cdot \frac{BE}{K} \tag{A2}$$

where E (Young's Modulus) = 11.8 GPa (Gammon and others, 1983); B (at -10°C) = $1.62 \times 10^{-7}\,s^{-1}\,MPa^{-3}$ (Barnes and others, 1971); K (rate of increase of K_1) = $0.01\,kPa\,m^{\frac{1}{2}}\,s^{-1}$; $K_c = 120\,kPa\,m^{\frac{1}{2}}$. This gives a value of r_{cr} of

$$r_{cr} = 5.26 \times 10^{-2}\,m.$$

Since the creep zone will grow from all around the notch, this result for the value of r_{cr} shows that at failure the whole of the region within the notch is within the creep zone (i.e. $2r_{cr} > D_n$).

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